Math 242, Topics in Algebra – Spring 2023

`https://sites.aub.edu.lb/kmakdisi/`

Problem set 7, due Tuesday, April 11 at the beginning of class

Exercises from Fraleigh:

Section 48, exercises 15, 18, 19, 22, 27, 32, 36.

Section 49, exercises 1, 2, 3, 10, 11, 13.

Additional Exercises (also required):

Exercise A7.1: Let F be a field, and $\overline{F}$ its algebraic closure.

a) If $\alpha, \beta \in \overline{F}$, show that α and β are F-conjugate if and only if there exists an (F-)automorphism $\sigma \in G(\overline{F}/F)$ such that $\sigma(\alpha) = \beta$.

b) Now take $F = \mathbf{Q}$, and let $\zeta = \exp(2\pi i/7)$ and $\alpha = \zeta + \zeta^{-1}$ (as in a previous exercise). Also review exercise 36 of Section 48. Use the action of $G(\mathbf{Q}(\zeta)/\mathbf{Q})$ on α to show that α has three $\mathbf{Q}$-conjugates: $\alpha_1 = \alpha$, α_2, and α_3. Use your expressions for these conjugates to compute the polynomial $g(x) = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) = \mathrm{irr}(\alpha, \mathbf{Q})$.

c) For which $\sigma \in G(\mathbf{Q}(\zeta)/\mathbf{Q})$ do we have $\sigma(\alpha) = \alpha$? More generally, over any field F and for any $\alpha \in \overline{F}$, why is the set $\{\sigma \in G(\overline{F}/F) \mid \sigma(\alpha) = \alpha\}$ a group?

Look at, but do not hand in, the following exercises:

Section 48, exercises 12, 14, 28, 33, 34, 35, 37, 38, 39.

Section 49, exercises 5, 6, 7, 8, 9.