Math 219, Linear Algebra I — Fall 2020
Course website: `https://sites.aub.edu.lb/kmakdisi/`
Problem set 7, due Thursday, November 5 at 2pm via Moodle

Exercises from Corwin-Szczarba:
Section 4.5, exercises 4, 6.
Section 4.6, exercises 1, 2, 5.
Section 4.7, exercises 1ab, 2, 8, 9.

Additional Exercises (also required):

Exercise A7.1: Let V be an inner product space, and let $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\}$ be a **basis** for V. Define $T : V \to \mathbf{R}^n$ by

$$T(\vec{\mathbf{v}}) = \begin{pmatrix} \langle \vec{\mathbf{v}}, \vec{\mathbf{v}}_1 \rangle \\ \langle \vec{\mathbf{v}}, \vec{\mathbf{v}}_2 \rangle \\ \vdots \\ \langle \vec{\mathbf{v}}, \vec{\mathbf{v}}_n \rangle \end{pmatrix}.$$

a) Show that T is a linear transformation.
b) Show that T is injective. (Hint: show that if $\vec{\mathbf{z}} \in \ker T$, then $\langle \vec{\mathbf{z}}, \vec{\mathbf{z}} \rangle = 0$.)
c) Show that T is surjective.
d) Deduce that for every subspace $W \subset V$ with $\dim W = \dim V - 1$, there exists a nonzero vector $\vec{\mathbf{w}}' \in W^{\perp}$. (Hint: choose a suitable basis for V, and apply the surjectivity result from part (c) in an appropriate way.)
e) In the situation of (d), show that $W^{\perp} = \operatorname{span}\{\vec{\mathbf{w}}'\}$. (This time, use injectivity of the T you used in (d).)
f) In $V = \mathcal{P}_3$, use the inner product $\langle f, g \rangle = \int_{x=0}^{1} f(x)g(x)\,dx$. Let $W = \mathcal{P}_2 \subset V$. Compute the subspace $W^{\perp}$ to illustrate the results of parts (d) and (e) above. (This computation can be carried out directly, even if you were not able to do other parts of this problem.)

Look at, but do not hand in:
Section 4.5, exercises 2, 9, 11.
Section 4.6, exercises 3, 4, 6, 7.
Section 4.7, exercises 1de, 6, 7, 10, 11, 12.
In Chapters 3 and 5, exercises 3.1.5, 3.1.6, 3.2.22, 3.3.13, 5.1.16.