Math 341, Modules and Rings I – Fall 2020
Course website: `https://sites.aub.edu.lb/kmakdisi/`
Problem set 6, due Tuesday, October 27 at 2pm via Moodle

Midterm exam: We will hold the midterm exam starting 8:15am on Monday, November 2, for the written component, followed by Tuesday, November 3 for the oral component. During the written part of the exam, everyone will be on webex with their cameras on. You write your answers on a paper, scan, and upload them as a single PDF. I plan to have the exam go from 8:15 to 9:45 on Monday morning, then you have 15 minutes to scan and upload your answers. As for the oral part on Tuesday, I will send you individual zoom links based on a questionnaire to know what times each of you is available.

Exercises from Jacobson, BA I:

Section 3.9, exercises 1, 2, 3.
Section 3.10, exercises 1, 2, 4, 7, 9 (see Exercise A6.2 first).

Additional Exercises (also required):

Exercise A6.1: (Taken from Fraleigh) Show that a finite abelian group is **not** cyclic if and only if it contains a subgroup isomorphic to $(\mathbf{Z}/p\mathbf{Z})^2$, for some prime p.

Exercise A6.2: Given a finite-dimensional F-vector space V and a linear transformation $T : V \to V$, show that T is diagonalizable if and only if when we view V as a module over $F[\lambda]$, then V is a direct sum of cyclic modules, each isomorphic to $F[\lambda]/\langle \lambda - a \rangle$. Under what conditions is V itself cyclic?

Exercise A6.3: Consider a $\mathbf{C}$-vector space V with a linear transformation $T : V \to V$, viewed as usual as a module over $R = \mathbf{C}[\lambda]$. We are given that, as an R-module, V is the direct sum of two cyclic modules:

$$V = \langle z_1 \rangle \oplus \langle z_2 \rangle, \qquad \text{with } \operatorname{Ann} z_1 = \langle (\lambda - 1)(\lambda^2 + 9) \rangle, \quad \operatorname{Ann} z_2 = \langle (\lambda - 1)^2 \rangle.$$

a) What are the characteristic polynomial and the minimal polynomial of T?
b) Give the rational canonical form matrix for T (using invariant factors).
c) Give the Jordan canonical form matrix for T (remember that the field is $\mathbf{C}$).

Look at, but do not hand in:

BA I, 3.9.4, 3.9.6, 3.9.7, 3.9.8, 3.9.9, 3.10.3, 3.10.6, 3.10.8, 3.10.10, 3.10.11, 3.10.12.