Math 219, Linear Algebra I — Fall 2020
Course website: `https://sites.aub.edu.lb/kmakdisi/`
Problem set 3, due Thursday, October 8 at 2pm via Moodle

Exercises from Corwin-Szczarba:

Section 2.5, exercises 10, 12. (The book says to solve exercise 12 by using exercise 10, but you may ignore their hint if you like — a direct proof is probably easier.)

Section 2.6, exercises 1abde, 2abdfi, 3abdej, 6, 8c, 15, 16. (Do 8c using the definition of matrix multiplication in terms of the entries in the matrices.)

Additional Exercises (also required):

Exercise A3.1: Given a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ with the following properties:

$$T(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad T(\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \qquad T(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

a) Find the matrix A_T of T.
b) Show that T is not injective.
c) Show that T is surjective.

Exercise A3.2: Define the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^3$ by

$$T(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}) = \begin{pmatrix} 3x_1 + 4x_2 \\ x_1 + 2x_2 \\ x_1 - x_2 \end{pmatrix}.$$

a) Find the matrix A_T.
b) Show that T is injective but not surjective.
c) Find **one** linear transformation $S : \mathbf{R}^3 \to \mathbf{R}^2$ such that $S \circ T = id_{\mathbf{R}^2}$ (there are many possible answers), and verify that $A_S A_T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Exercise A3.3: Given linear transformations $T : V \to W$ and $S_1, S_2 : W \to Z$. Assume that T is a surjection.
a) Show that $\text{Image}\,(S_1 \circ T) = \text{Image}\, S_1$.
b) (unrelated to part (a)) Assume that $S_1 \circ T = S_2 \circ T$. Show that $S_1 = S_2$.

Look at, but do not hand in, the following exercises:

Section 2.6, exercises 5, 7, 8 (all parts: do via the entries of the matrices and also in terms of linear transformations), 9, 10, 13, 14, 17.