Math 241, Introduction to Abstract Algebra – Fall 2019
Course website: `https://sites.aub.edu.lb/kmakdisi/`
Problem set 3, due Wednesday, September 25 at the beginning of class

Exercises from Fraleigh:

Section 6, exercises 53, 56. (These require Exercise A3.1. Hint for exercise 56 part a: if $H = \langle h \rangle$ and $K = \langle k \rangle$, what can you say about the element $hk \in G$? As for exercise 56 part b, one possible hint is to do exercise A3.2 first. Another possible hint is to think about prime factorizations and to use the previous exercise 53.)

Section 8, exercises 1, 4, 5 (also express both τ and $\sigma^{-1}\tau\sigma$ as products of cycles), 6, 7, 8, 21, 46.

Section 9, exercises 7, 10, 13.

Additional Exercises (also required):

Exercise A3.1: Let $a, b \in \mathbf{Z}$, and assume that $GCD(a, b) = 1$.

a) Show that there exist $x_0, y_0 \in \mathbf{Z}$ such that $ax_0 + by_0 = 1$.

b) Suppose $m \in \mathbf{Z}$ and $a|bm$. Show that $a|m$. (Hint: multiply the equation in part (a) by the number m.)

c) Unrelated to part (b): suppose $(G, \cdot)$ is an abelian group, and $g, h \in G$ have orders a and b respectively. What can you say about $(gh)^{ax_0}$? Where did you use commutativity?

Exercise A3.2: Let $a, b \in \mathbf{Z}^+$.

a) Show that $a\mathbf{Z} \cap b\mathbf{Z}$ is a subgroup of $\mathbf{Z}$, not equal to $\{0\}$.

b) We thus know that there exists $m > 0$ such that $a\mathbf{Z} \cap b\mathbf{Z} = m\mathbf{Z}$. Explain why m is the least common multiple of a and b. We write $m = LCM(a, b)$.

c) Also explain why if c is any common multiple of a and b, then c is a multiple of m.

d) If $d = GCD(a, b)$ with $a = da'$, $b = db'$ and $GCD(a', b') = 1$, show that $LCM(a, b) = da'b' = ab/GCD(a, b)$. (Hint: what can you deduce if $da'x = db'y$? Why is this relevant to common multiples of a and b?)

Look at, but do not hand in:

Section 6, exercise 54.

Section 8, exercises 16, 30–34, 40–43, 44, 45, 47.

Section 9, exercises 30, 34, 36, 37, 39.