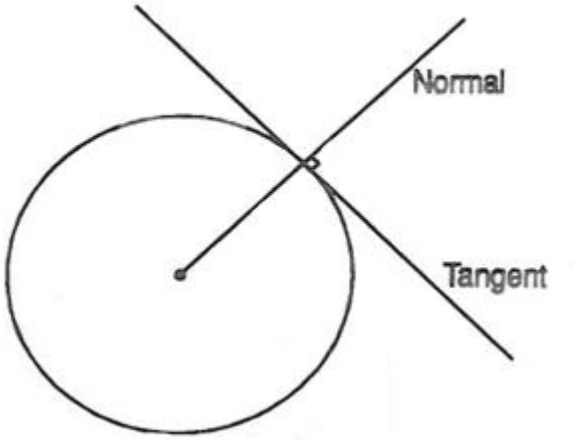
TANGENTS AND NORMAL'S

The following diagram shows the tangent and normal to a circle at a point on its circumference.



Both the tangent and normal are straight lines of the form $y = mx + c$ and to find these lines we need two pieces of information, the gradient of the line and a point that the line passes through.

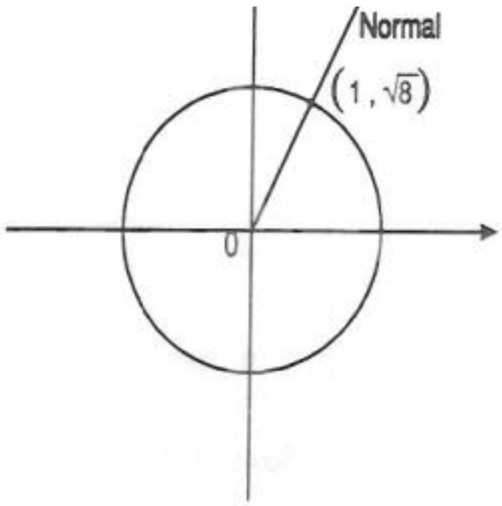
The normal to a circle will always pass through the centre of the circle.

The tangent and the normal are always at right angles to each other.

Example

Find the equation of the normal circle $x^2 + y^2 = 9$ at the point (1, $\sqrt{8}$) on its circumference.

By examining the equation of the circle you can see that the centre point is
(0, 0).



To find the gradients use $\frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are two points on the line.

So the gradient = $\frac{\sqrt{8}}{1}\frac{-0}{0} = \frac{\sqrt{8}}{1} = \sqrt{8}$ = M

So I've found one piece of information, now I need to use $y - y_1 = M(x - x_1)$ to find the equation of the line.

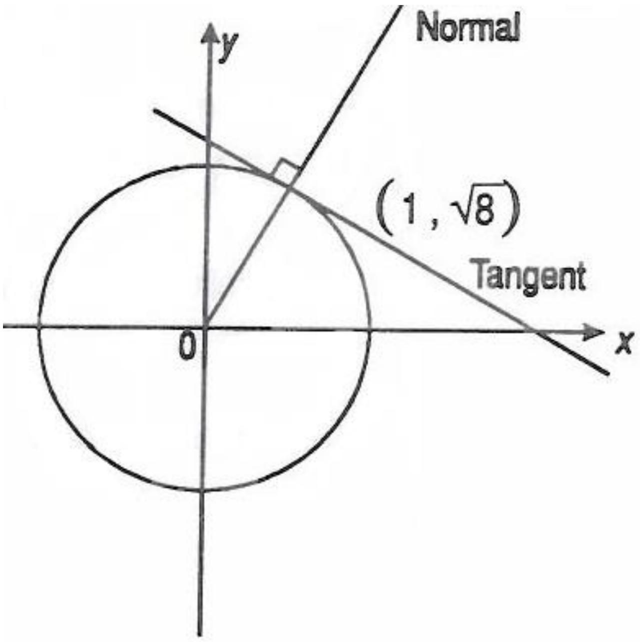
I'll use the point (0, 0). I could use (1, $\sqrt{8}$) but I think (0, 0) would be easier!

So $y - 0 = \sqrt{8}\ (x - 0)$

So $y = \sqrt{8}\,x$ This is the equation of Normal.

Example

In the example find the equation of the tangent.



I know the tangent is at right angles to the normal so that the product of their gradient is -1.

So gradient of a tangent = $-\frac{1}{\sqrt{8}}$

So now I know the gradient, I need to take a point on the line. The only one I know is the point (1, $\sqrt{8}$)

So $y - \sqrt{8} = -\frac{1}{\sqrt{8}}(x - 1)$

$y - \sqrt{8} = -\frac{x}{\sqrt{8}} + \frac{1}{\sqrt{8}}$

$\sqrt{8}y - 8 = -x + 1$ (multiply through by $\sqrt{8}$)

$\sqrt{8}y + x = 9$ Equation of the tangent.

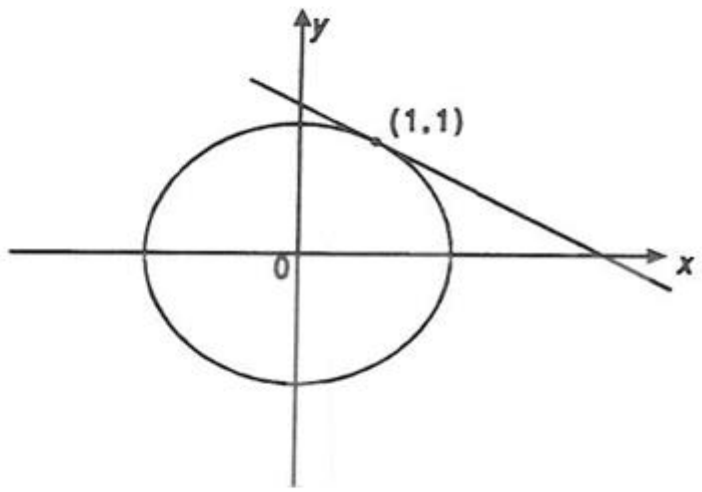
This may seem complicated but all we really need is the gradient of these lines and a point that they pass through.

Example

Find the equation of the tangent to the circle $2x^2 + 2y^2 = 4$ at the point (1, 1).

Dividing by 2 gives $x^2 + y^2 = 2$

So the centre of the circle is (0,0)



So the gradient of the normal is $\frac{1-0}{1-0} = 1$

Hence the gradient of the tangent is -1 (Remember product of gradient is -1)

Using the point (1, 1) the equation of the tangent is

$y - 1 = -1(x - 1)$

$y - 1 = -x + 1$

$y = -x + 2$

Exercise 1

Find the equation of the normal to the circle whose equation is

$(x - 1)^2 + (y - 1)^2 = 2$ at the point (2, 2).

Now check your answers.

Exercise 2

Find the equation of the tangent from the previous activity.

Now check your answer

Exercise 3

Find the equation of the tangent and normal to the circle with the equation
$2y^2 + 2x^2 + 8x - 2y - 34 = 0$ at the point (3, 4).

Now check your answers

SUMMARY

1. Equation of a circle is:

   $(x - a)^2 + (y - b)^2 = r^2$

   Where: the centre is (a, b) and the radius = r

   Note: The coefficient of x and y must be the same;

   r^2 must be positive.

2. An alternative form of equation is:

   $x^2 + y^2 - 2ax - 2ay + a^2 + b^2 - r^2 = 0$

   Note: There is no xy term

3. The tangent and normal to a circle are always at right-angles to each other so the product of their gradients is -1

ANSWERS

Exercise 1

The centre is (1, 1)

The gradient of the normal is $\frac{2-1}{2-1} = 1$

Using the centre of the circle the equation of the normal is:

$y - 1 = 1\,(x - 1)$

$y - 1 = x - 1$

$y = x$

Now return to the text

Exercise 2

The equation of the normal is $y = x$

So the gradient of the normal is 1

The gradient of the tangent is -1 (product of gradients)

Using the point (2, 2)

The equation of the tangent is

$y - 2 = -1\,(x - 2)$

$y - 2 = -x + 2$

$y = x + 4$

Now return to the text

Exercise 3

First we need to find the centre of the circle

So divide by 2 $x^2 + y^2 + 4x - y - 17 = 0$

Complete the square

$(x + 2)^2 - 4 + (y - \frac{1}{2})^2 - \frac{1}{4} - 17 = o$

$(x + 2)^2 + (y - \frac{1}{2})^2 = 21\frac{1}{4}$

So the circle has centre $(-2, \frac{1}{2})$

Now the gradient of the normal.

Gradient of normal $= \frac{4 - \frac{1}{2}}{3 - (-2)} = \frac{3\frac{1}{2}}{5} = \frac{7}{10}$

$\therefore$ Equation of normal is $y - 4 = \frac{7}{10}(x - 3)$

$y - 4 = \frac{7x}{10} - \frac{21}{10}$

$10y - 40 = 7x - 21$

Equation of normal: $10y - 7x = 19$

Now for the tangent:

Gradient of tangent is $-\frac{10}{7}$

Equation of tangent is $y - 4 = -\frac{10}{7}(x - 3)$

$y - 4 = -\frac{10}{7} + \frac{30}{7}$

$7y - 28 = -10x + 30$

Equation of tangent: $7y + 10x = 58$

Now return to the text