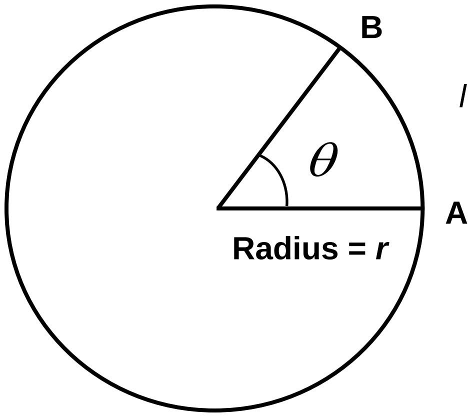
RADIANS

RADIAN MEASURE

We have seen that an angle is usually measured in degrees but there is another way of measuring an angle. This is known as the radian (abbreviation rad).



l = Length of arc AB

$$\text{Angle in radians} = \frac{\text{Length of arc}}{\text{Radius of circle}}$$

$$\theta \text{ radians} = \frac{l}{r}$$

RELATION BETWEEN RADIANS AND DEGREES

If we make the arc AB equal to a semi-circle then

Length of arc = π r

And Angle in radians = $\frac{\pi r}{r}$ =r

Now the angle subtended by a semi-circle = 180^0

Therefore π radians = 180^0

Or 1 radian = $\frac{180°}{r}$ = 57.3^0

Thus to convert from degrees to radians

$\theta^\circ = \frac{\pi\theta}{180}$ radians

Thus 30º = $\frac{\pi(30)}{180}$ rad = $\frac{\pi}{6}$ rad

90º = $\frac{\pi}{2}$ rad   180º = π rad

45^0 = $\frac{\pi}{4}$ rad   270º = $\frac{3\pi}{2}$ rad

60º = $\frac{\pi}{3}$ rad   360º = 2 π rad

To convert from radians to degrees

θ radians $= (\frac{180}{\pi} x\theta)^{\circ}$

Example 1

Convert 29º37’(min) 29”(sec) to radians stating the answer correct to 4 significant figures.

The first step is to convert the given angle into degrees and

29º37’29” = 29 + $\frac{37}{60}$ + $\frac{29}{3600}$ = 29.625^0

= $\frac{\pi \times 29.625}{180}$ = 0.5171 radians

Many scientific calculators will convert degrees, minutes and seconds into decimal degrees, and vice versa, using special keys.

Example 2

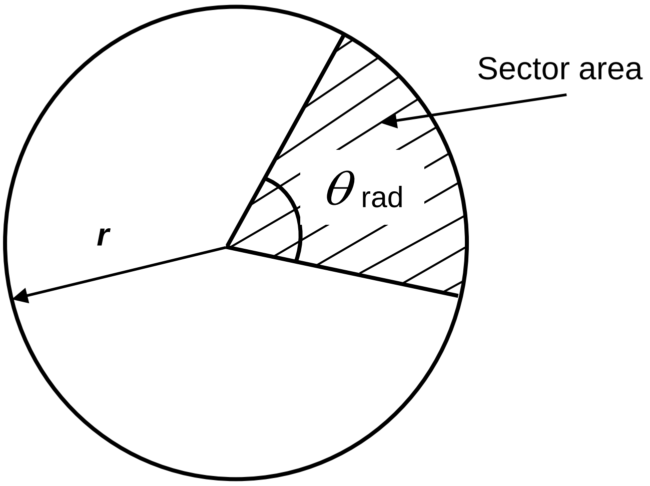
Convert 0.08935 radians into degrees, minutes and seconds.

0.08935 radians = $\frac{0.08935 \times 180}{\pi}$ = 5.1194^0 = 5º7’10”

THE AREA OF A SECTOR

The area of a circle = πr^2
So, by proportion, referring to the figure below gives

Area of sector = $\pi r^2 \times \dfrac{\theta}{2\pi} = \dfrac{1}{2} r^2 \theta$



Example 3

Find the angle of a sector of radius 35mm and area 1020mm^2

Now Area of sector = $\dfrac{1}{2} r^2 \theta$

And substituting the given value of

Area = 1020mm^2 and r = 35mm

We have $1020 = \dfrac{1}{2}(35)^2 \theta$

From which $\theta = \dfrac{1020 \times 2}{35^2}$ = 1.67rad

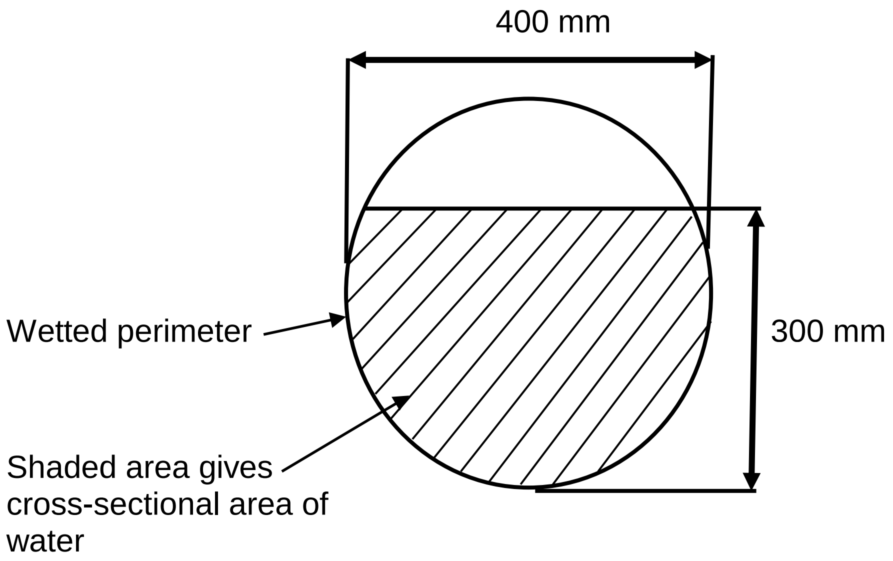
$= \dfrac{180 \times 1.67}{\pi}$ = 95.7^o

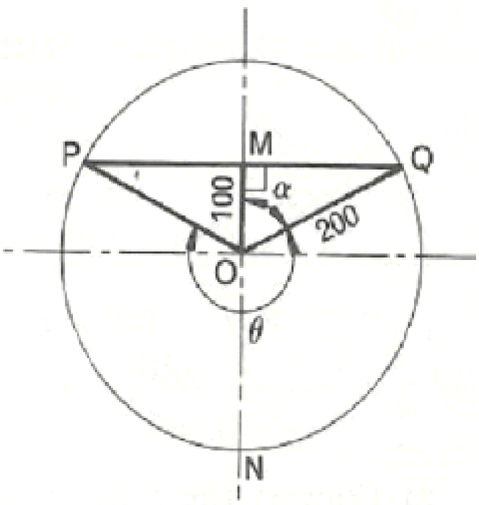
Summary

Length of arc of sector $= r\theta$ or $2\pi r\left(\dfrac{\theta^0}{360}\right)$
Area of sector $\dfrac{1}{2} r^2 \theta$ or $\pi r^2\left(\dfrac{\theta^0}{360}\right)$

Example 4

Water flows in a 400mm diameter pipe to a depth of 300mm. Calculate the wetted perimeter of the pipe and the area of cross-section of the water.





The right-angled triangle MQO

$$\cos\alpha = \frac{OM}{OQ} = \frac{100}{200} = 0.5$$

Also $\quad \sin\alpha = \dfrac{MQ}{OQ}$

$\therefore \quad$ MQ = OQ sin α = 200 sin 60° = 173.2mm

Now $\quad \theta + 2\alpha = 360^o$

$\therefore \quad \theta = 360^0 - 2(60^o) = 240^o$

Thus
Wetted perimeter = Arc PNQ

$$=2\pi r\left(\frac{\theta^0}{360}\right) = 2\pi(200)\left(\frac{240}{360}\right) = 838\text{mm}$$

Also

(Cross-sectional) = (Area of) + (Area of)
(area of water) (sector PNG) (triangle PDG)

$$= \pi r^2\left(\frac{\theta^0}{360}\right) + \frac{1}{2}\ (PQ)\ (MO)$$

$$= \pi(200)^2\left(\frac{240}{360}\right) + \frac{1}{2}(2 \times 173.2)(100)$$

$= 83780 + 17320$
$= 101OOO\text{mm}^2$

Exercise 1

Convert the following angles to radians stating the answers correct to 4 significant figures:

a) 35^o b) $83^o28'$ c) $19^o17'32''$ d) $43^0 39'49''$

Exercise 2

Convert the following angles to degrees, minutes and seconds correct to the nearest second:

a) 0.1732 radians b) 1.5632 radians c) 0.0783 radians

Exercise 3

If r is the radius and θ is the angle subtended by an arc, find the length of arc when:

a) $r = 2\text{m}$, $\theta = 30^0$ b) $r = 34\text{mm}$, $\theta = 38^o40'$

Exercise 4

If l is the length of an arc, r is the radius and θ the angle subtended by the arc, find θ when:

a) $l = 9.4\text{mm}$, $r = 4.5\text{mm}$ b) $l = 14\text{mm}$, $r = 79\text{mm}$

Exercise 5

If an arc 70mm long subtends and angle of 45^0 at the centre, what is the radius of the circle?

Exercise 6

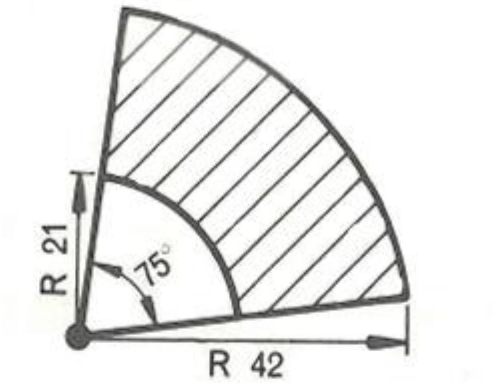
Find the area of the following sectors of circles:

a) radius 3m, angle of sector 60^o b) radius 27mm, angle of sector $79^o45'$

c) radius 78mm, angle of sector $143^{o}42'$

Exercise 7

Calculate the area of the part shaded:



Exercise 8

A chord 26mm is drawn in a circle of 35mm diameter. What are the lengths of arcs into which the circumference is divided?

Exercise 9

The radius of a circle is 60mm. A chord is drawn 40mm from the centre. Find the area of the minor segment.

Exercise 10

In a circle of radius 30mm a chord is drawn which subtends an angle of 80^{o} at the centre. What is the area of the minor segment?

Exercise 11

A flat is machined on a circular bar of 15mm diameter, the maximum depth of cut being 2mm. Find the area of the cross section of the finished bar.

Exercise 12

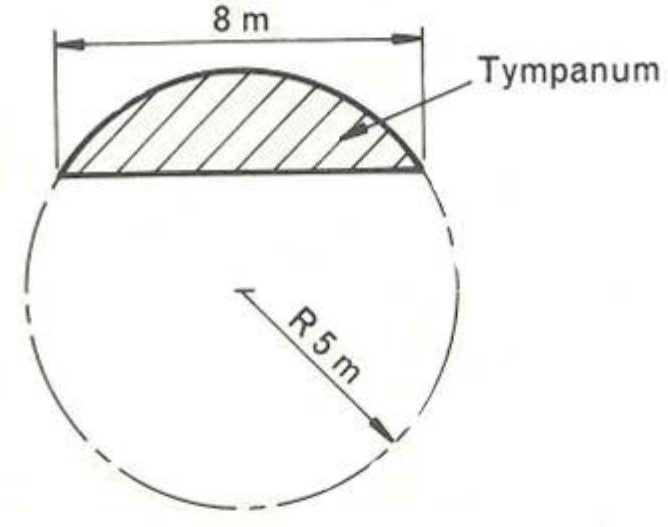
Water flows in a 300mm diameter drain to a depth of 200mm. Calculate the wetted perimeter of the drain and the area of the cross section of the water.

Exercise 13

In marking out the plan of part of a building, a line 8m long is pegged down at one end. Then with the line held horizontal and taut, the free end of is swung through an angle of 57^{o}. Calculate the distance moved by the free end of the line and determine the area swept out.

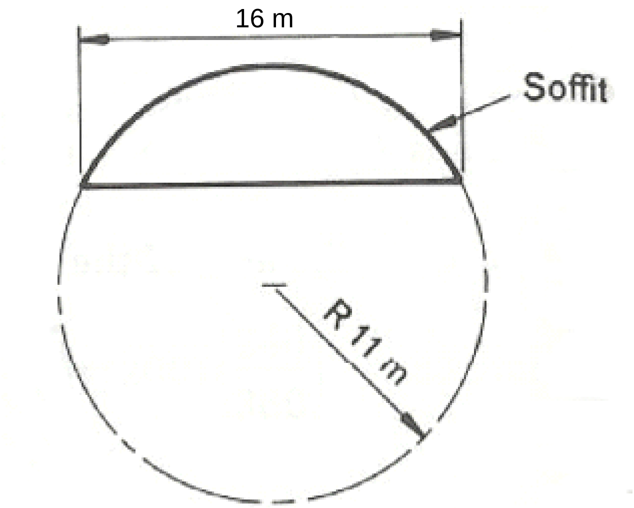
Exercise 14

Find the area of the brickwork necessary to fill the tympanum of the segmental arc shown.



Exercise 15

Below shows a segmental arch for a bridge.   Calculate the length of the soffit of the arch.



ANSWERS

Exercise 1

a) 0.6108 b) 1.457 c) 0.3367 d) 0.7621

Exercise 2

a) $9^o55'25''$ b) $89^o33'53''$ c) $4^o29'11''$

Exercise 3

a) 1.05m b) 22.9mm

Exercise 4

a) 120^o b) 10.2^o

Exercise 5

89.2mm

Exercise 6

a) $4.71m^2$ b) $508mm^2$ c) $7620mm^2$

Exercise 7

$866mm^2$

Exercise 8

29.3 and 80.7 mm

Exercise 9

$1240mm^2$

Exercise 10

$185mm^2$

Exercise 11

$163mm^2$

Exercise 12

369mm, $20600mm^2$

Exercise 13

7.96m, $31.7m^2$

Exercise 14

$11.2m^2$

Exercise 15

17.9m