Learning Development

University of Northampton

CONSTRUCTIONS AND LOCI.

Constructions and Scale drawing occur frequently on examination papers.
All you have to do is to follow the steps shown below for the various constructions.

Do not worry if you do not "get it right" the first time-like lots of things, you need to practise to be able to do something well!

Make sure that you have a **sharp** pencil, a rubber, a ruler, a protractor, a pair of compasses (with a short pencil) and a set square.

If you have not used this equipment before, please ask a member of staff to help. Once you have mastered the basic steps, constructions are easy and very pleasant to do! Why not have a go, drawing circles, and arcs, before you start, just so that you know how to hold the instruments?

There are 4 basic constructions:

1. Constructing a given triangle.

2. Cutting a line in two equal parts - this forms a right angle, or two perpendicular lines.

3. Halving an angle - bisecting an angle.

4. Making a 60^0 angle.

When you can do these few simple constructions, you can do all of the constructions necessary in the examination.

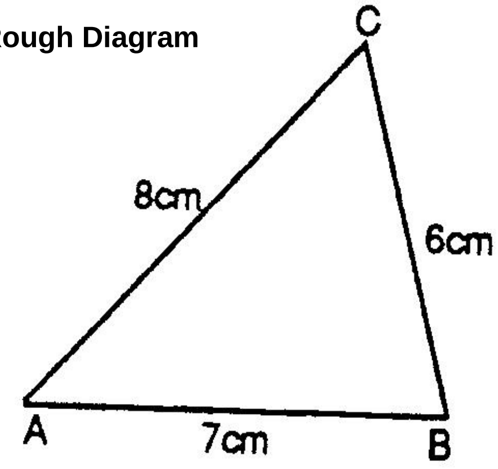
READY! Now carry out all the following constructions doing each step as instructed.

First always draw a **rough diagram** of the information you are given.

1. i) To construct a given triangle.

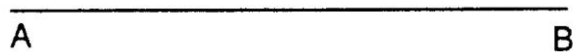
Question - Construct a triangle ABC with AB = 7cm, BC = 6 cm and AC = 8 cm.

Rough Diagram

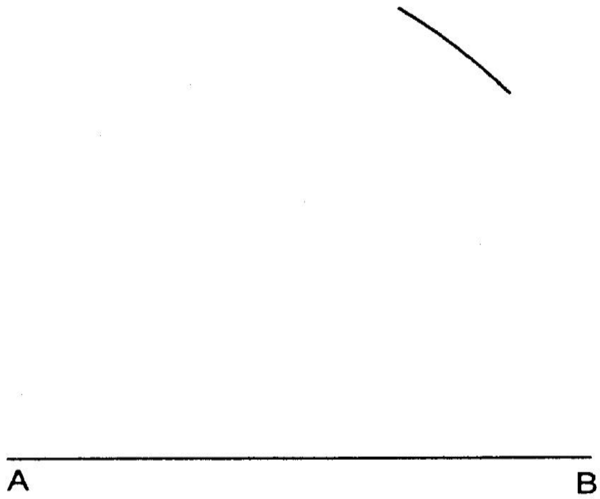


Add all the measurements, which you are given in the question to the diagram.

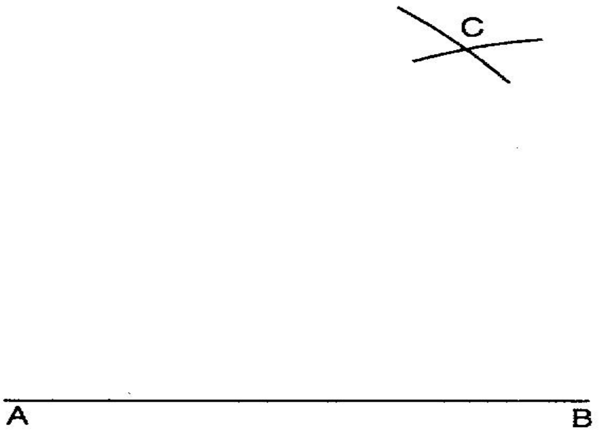
a) Draw a base line (it is easier to draw a horizontal line). Measure a length of 7 cm. Call this AB.



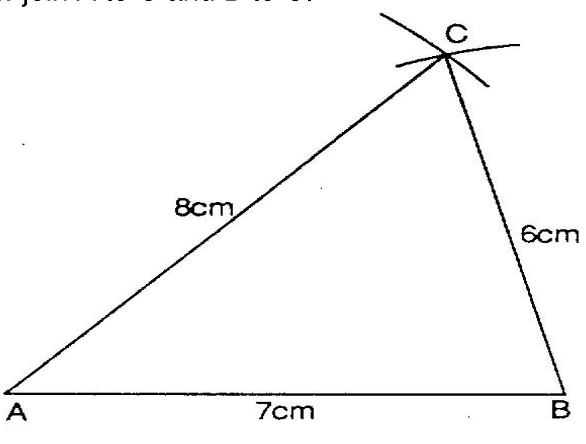
b) Put your compass point on A and open the compasses so that the distance between the point and the pencil is 8cm. Draw an arc.



c) Now put your compass point on B and open the compasses so that the distance between the point and the pencil is 6 cm. Draw an arc. The arcs should cross. **If they do not cross**, you must make the arcs longer.

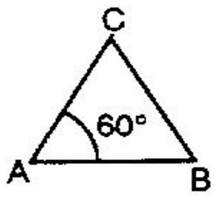


Now join A to C and B to C.



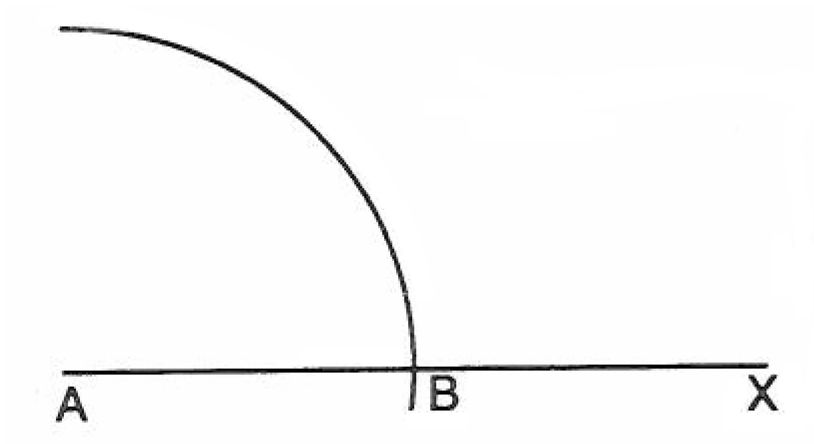
ii) To construct an equilateral triangle.

Rough Diagram

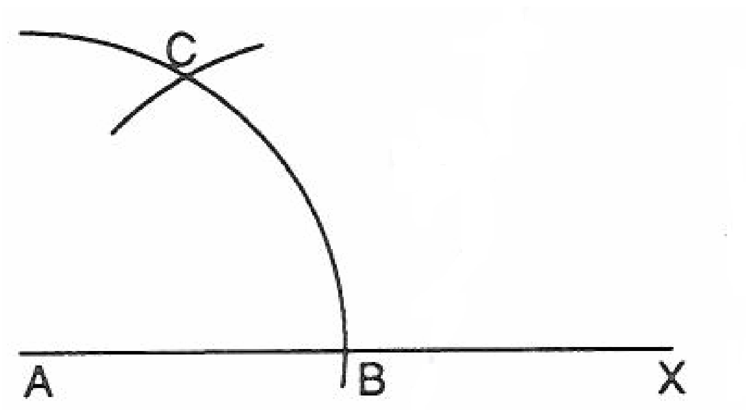


a) Draw a line AX.

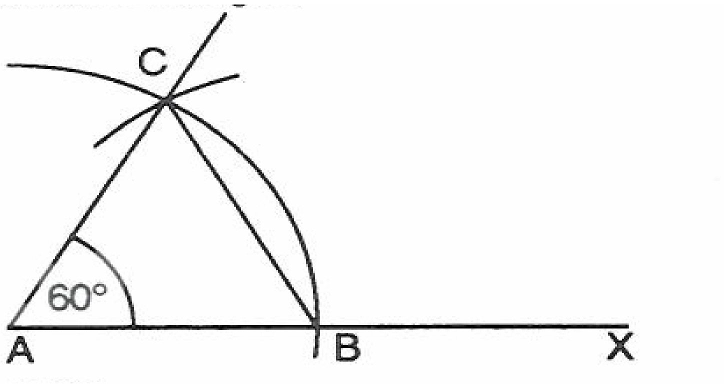
b) Put your compass point on A, and draw an arc **(ANY RADIUS)**. Let it cut AX at B. Do not alter the distance between the compass point and pencil (the radius).



c) Now put your compass point on B and keeping the same radius, draw a second arc. Both arcs should cross. Call the point where they cross point C.



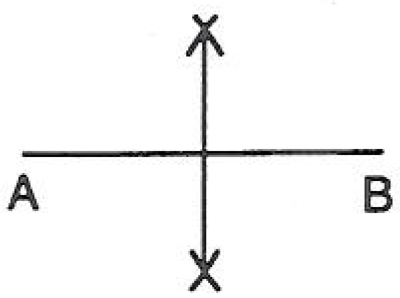
d) Join the points to give AC. The angle BAC you have constructed is 60^0. (You have constructed part of an equilateral triangle). By joining the points BC, we have constructed an equilateral triangle.



2. i) To divide a line into 2 equal parts (called bisecting a line).

Your finished construction should look like this.

Rough Diagram

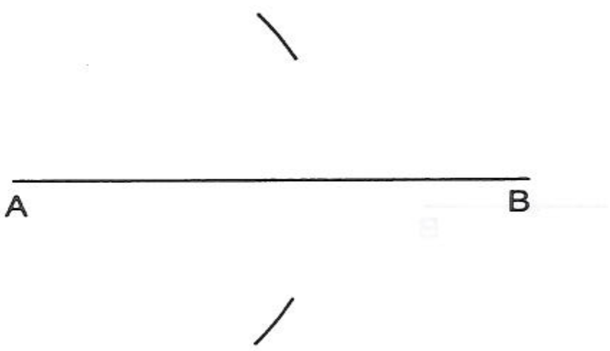


a) Draw a line and label the ends A, B.

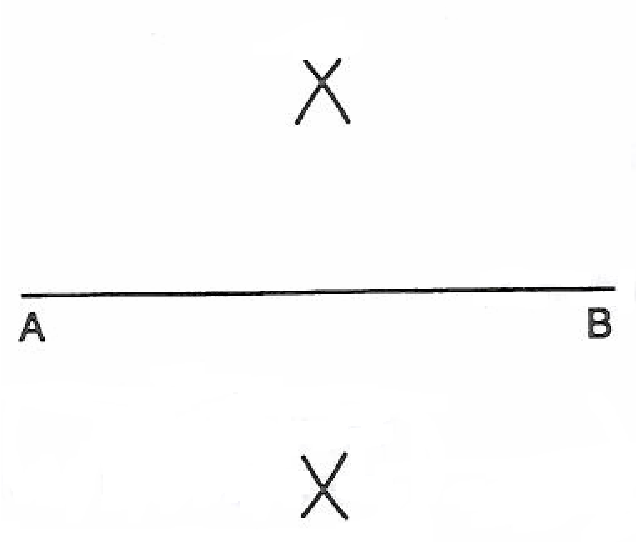
The line can be any length you wish.

b) Open your compasses so that the radius (distance between the pencil point and the point of the compasses) is **greater** than half the length of the line.

c) Put your compass point on A, and draw an arc above and below the line. **Do not alter the distance between the compass point and the pencil.**

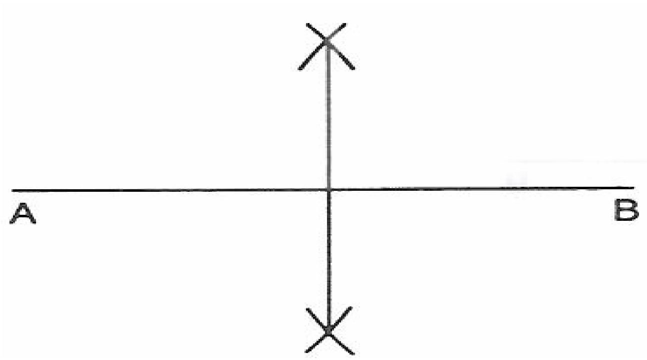


d) With the distance between the pencil and the compass point still the same, now put the compass on B, and again, draw an arc above and below the line AB.



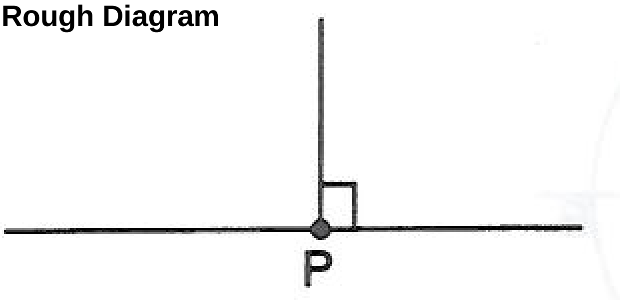
e) The arcs should cross above and below the line. **If they do not, do not worry. It simply means that you have not chosen a big enough radius!** Go back to the beginning, and make sure that the radius is BIGGER THAN half of the line you are given.

e) Line up your ruler, with the points made by the arcs above and below the line. Draw the line. You should obtain a line which is half way between A and B.



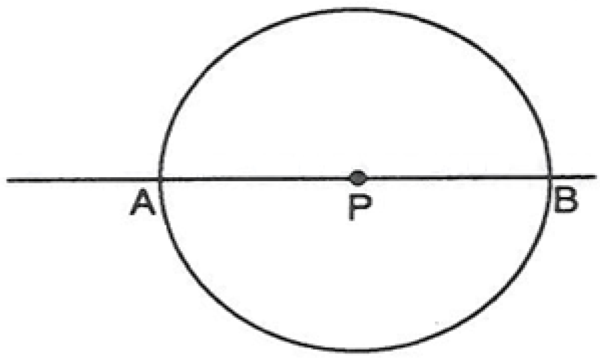
ii) To draw a perpendicular at a given point P on a straight line. (A perpendicular is a line, which crosses another line at 90^0 i.e. at right angles).

Rough Diagram

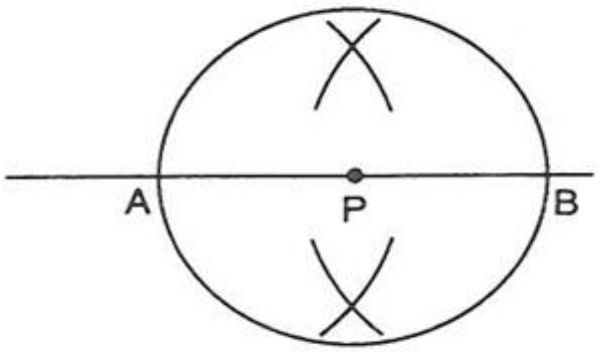


a) Draw a line and label the point P as shown.

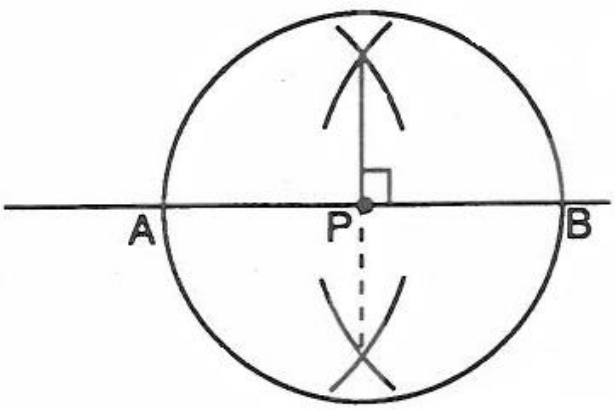
b) Put your compass point on P and draw a circle, of **any** radius, so at it crosses the line. Label where it crosses A and B.



c) **Now do exactly what you did in Construction 2(i)**, using AB as the line you are bisecting (or cutting into two equal parts).

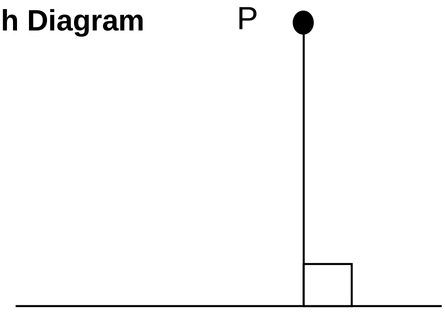


d) Join up the points made by the arcs, and the line should pass through P. You have constructed a perpendicular at a point P!

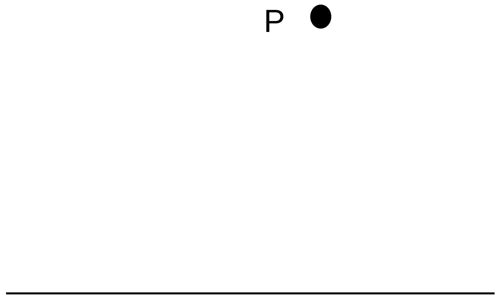


iii) To draw the perpendicular to a line from a given point P which is not on the line.

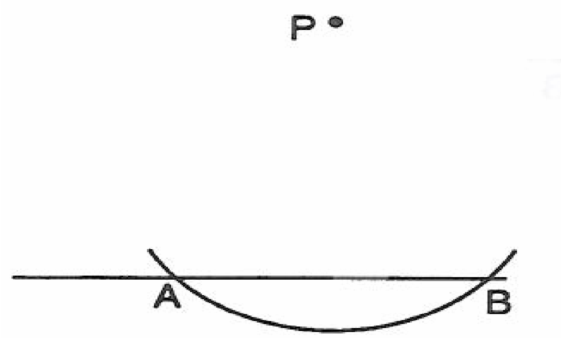
Rough Diagram



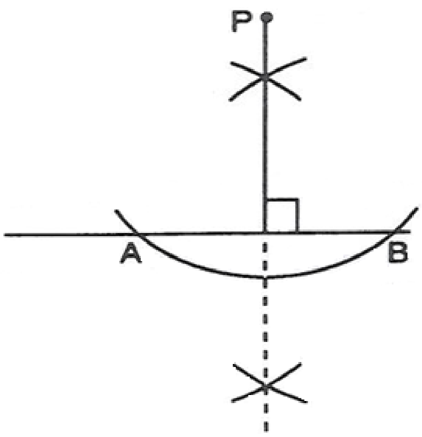
a) Draw a line and label the point P, not on the line, as shown above.



b) Put your compass point on P, and draw an arc to cut the line at two-point A and B.



c) **Now do exactly what you did in Construction 2(i)**, using AB as the line you are bisecting.

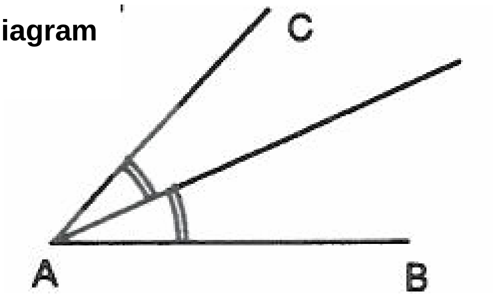


d) Join up the points made by the arcs, and the line you draw should go through point P.

N.B CONSTRUCTIONS 2 i), ii) AND iii) ARE ALL DIFFERENT WAYS OF CONSTRUCTING TWO PERPENDICULAR LINES!

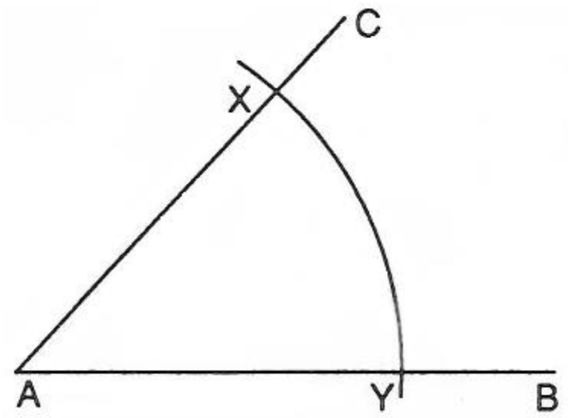
3. To bisect a given angle.

Rough Diagram

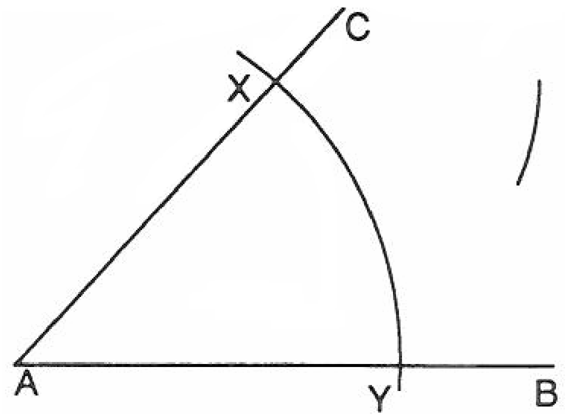


a) Draw any angle without a protractor. Label this CAB as above.

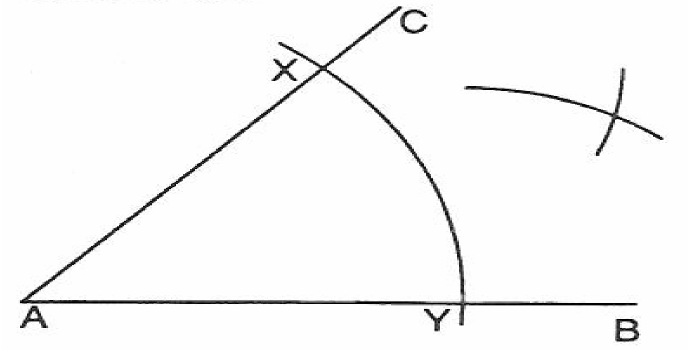
b) Put your compass point on A and draw an arc. Let it cross the lines AC and AB at X and Y respectively.



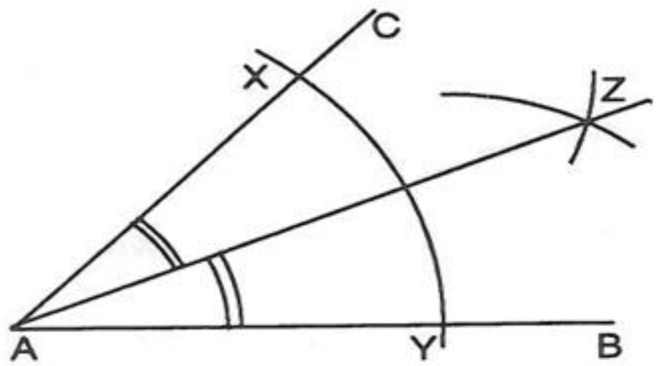
c) Open your compass so that the distance between the point and the pencil (the radius) is greater than half the distance between XY. Put the compass point on X and draw an arc.



d) Put the compass point on Y, keeping the same radius, and draw another arc.



e) The arcs should cross (at Z). Join points to give the line AZ. This line bisects the angle at CAB.

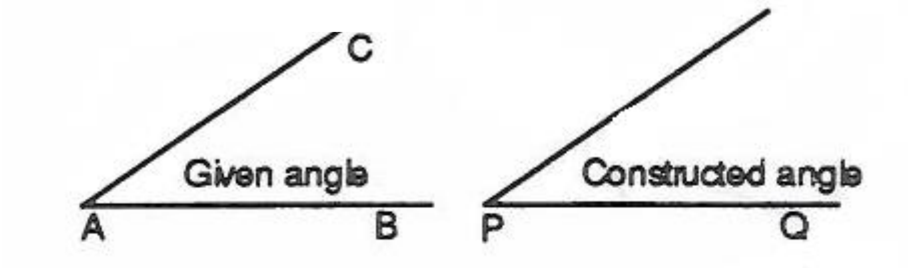


N.B

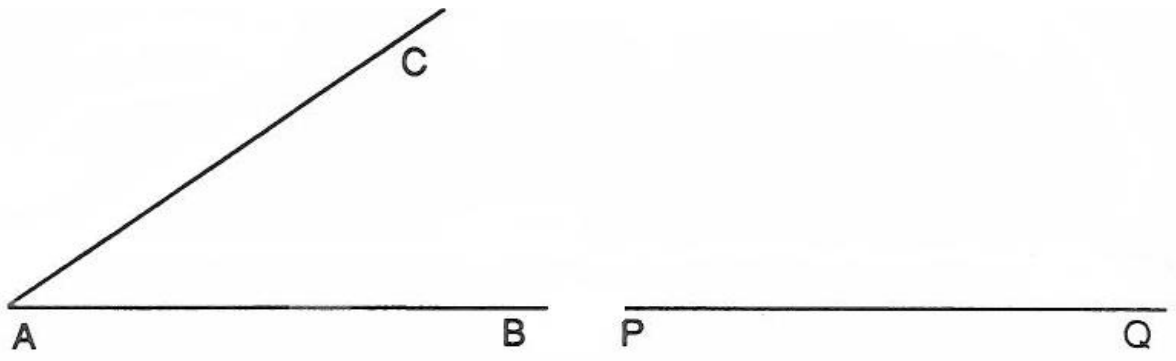
- To construct an angle 45^0, FIRST, construct a perpendicular (90^0) and THEN bisect the angle.
- To construct an angle of 30^0, FIRST, construct an angle of 60^0 and THEN bisect the angle.

4. To construct an angle equal to a given angle CAB.

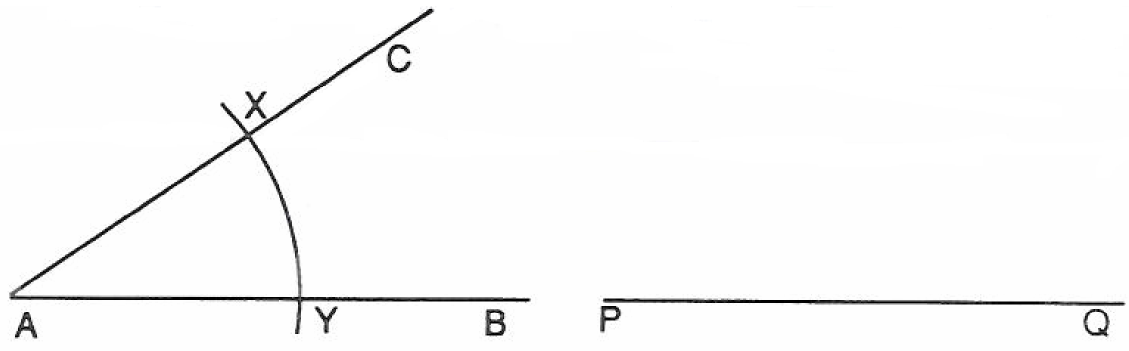
Rough Diagram



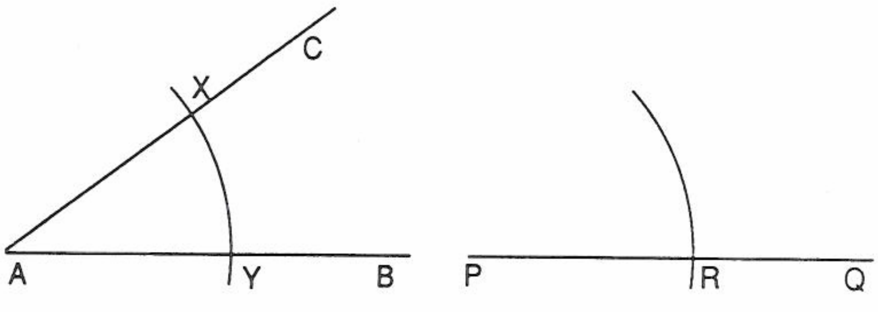
a) Draw the given angle CAB. Draw the line PQ as shown.



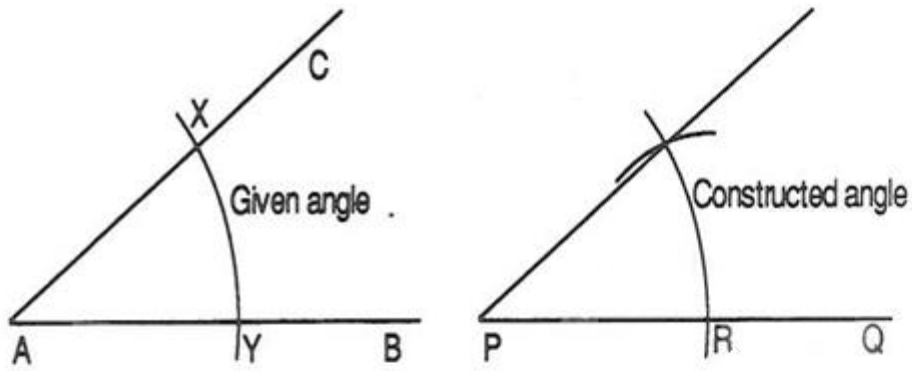
b) Put your compass point on A and draw an arc. Let it cross the lines AC and AB at X and Y respectively.



c) Keeping the same radius put the compass point on P and draw an arc. Let it cross PQ at R.

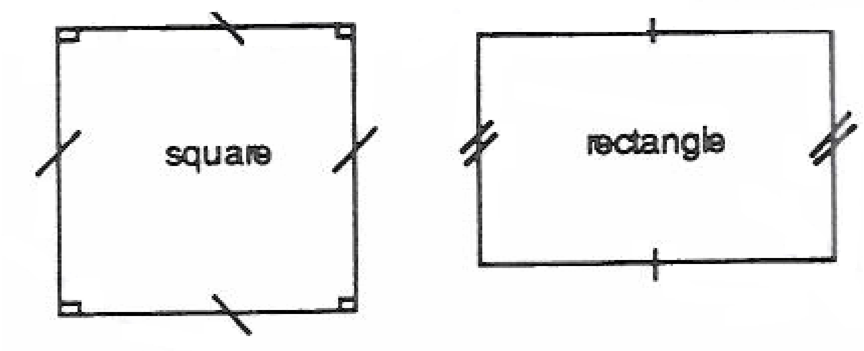


d) Now open your compasses to give you the distance between X and Y. Keeping this radius, now put your compass point on R, and make an arc which crosses the one you made in step (c). Join this point with P. You have drawn an angle equal to the one given in the first diagram.



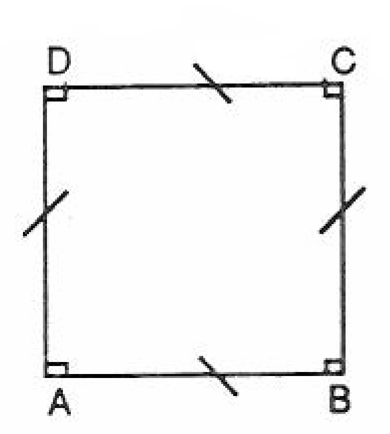
THE CONSTRUCTION OF A QUADRILATERAL

Rough diagram of various quadrilaterals



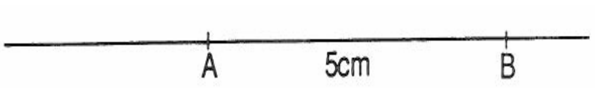
SQUARE

Draw a 5cm square.

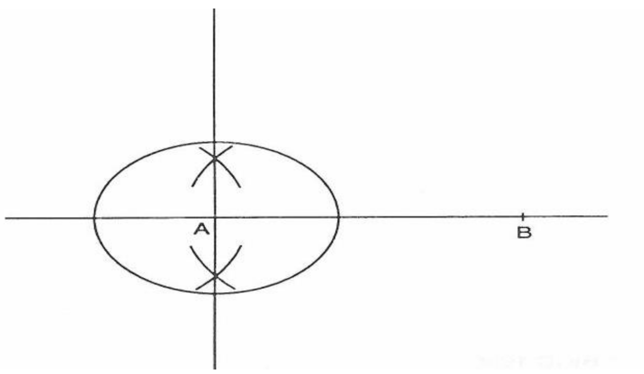


Rough Diagram

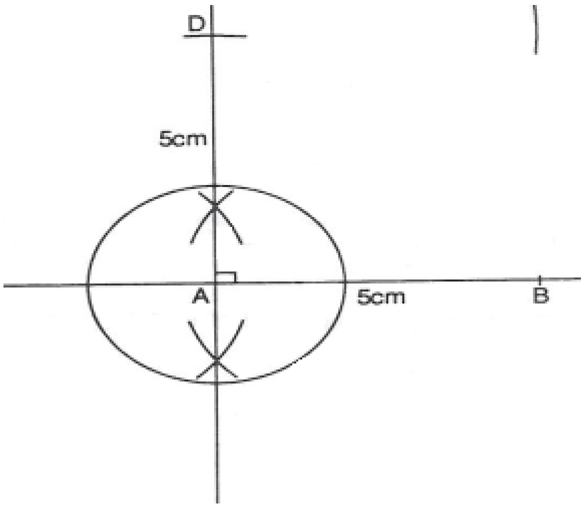
a) Draw a line of any length, as a base line. On this mark the actual measurement of the line AB.



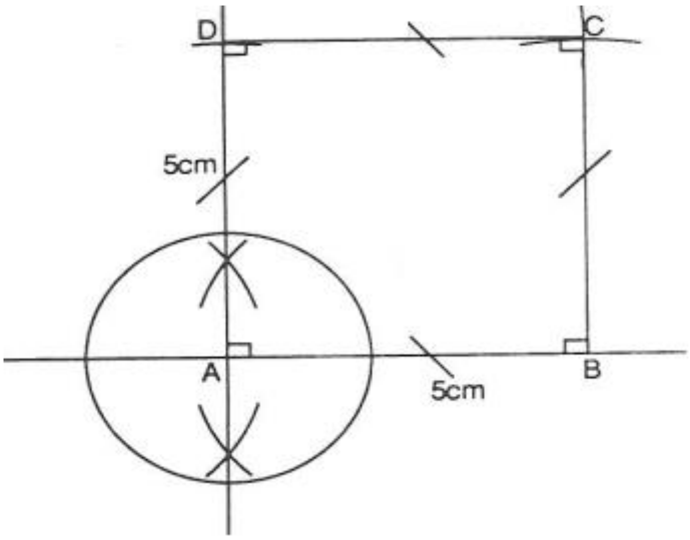
b) Construct the perpendicular at point A (see Construction 4).



c) Take your compasses and open them up to 5 cm, and put the point of the compasses on A, making a mark on the perpendicular line. Mark this point D.
Leave your compasses open at 5 cm. Put your compass point on D and mark another arc.



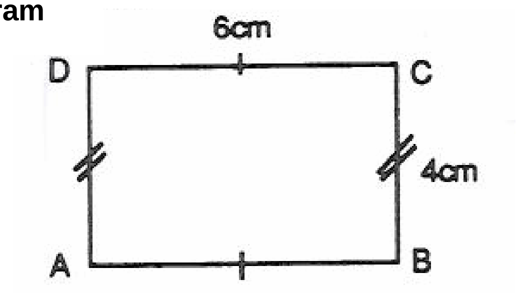
With your compasses still open at 5 cm, put the point on B and draw another arc, which should cross the one from D. This gives you point C. Now connect the points B to C, and C to D- and you have drawn the square.



RECTANGLE

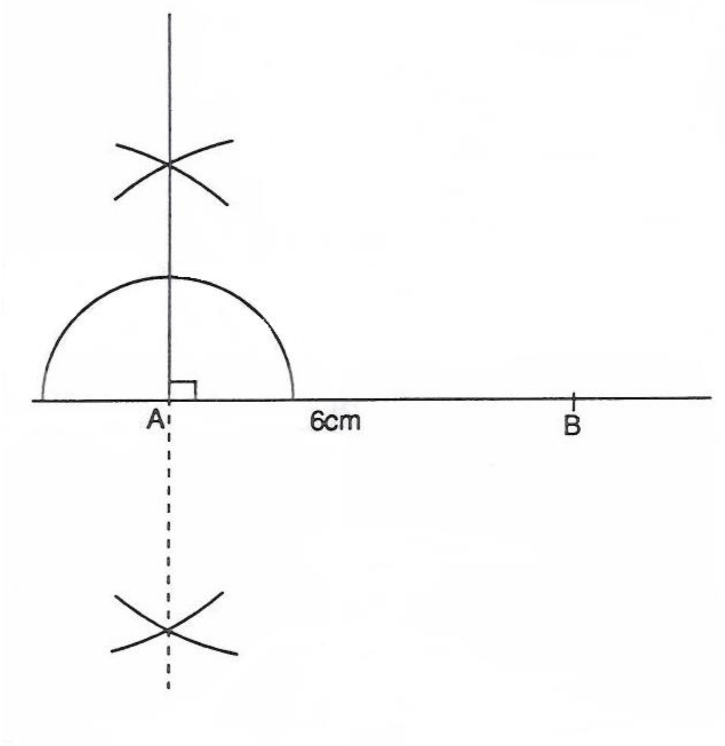
Draw a rectangle measuring 4 cm x 4 cm.

Rough Diagram

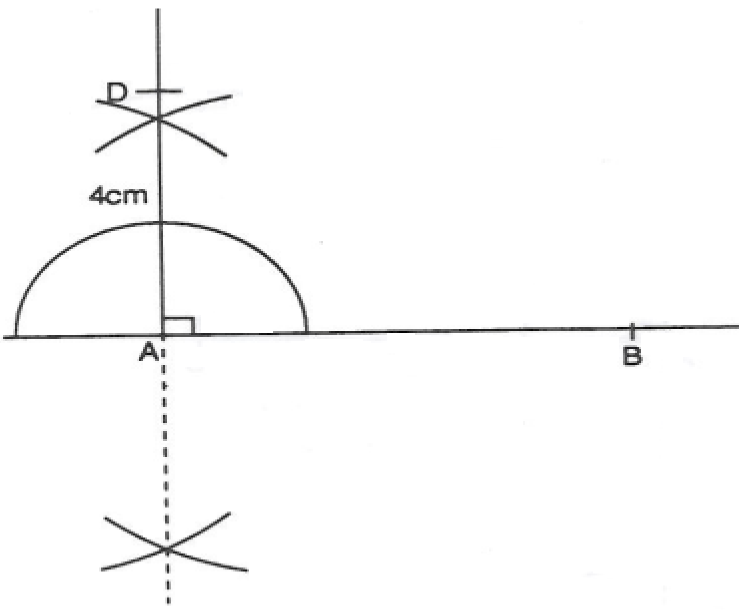


a) Draw a line across your page, then mark off the measurement AB = 6 cm.

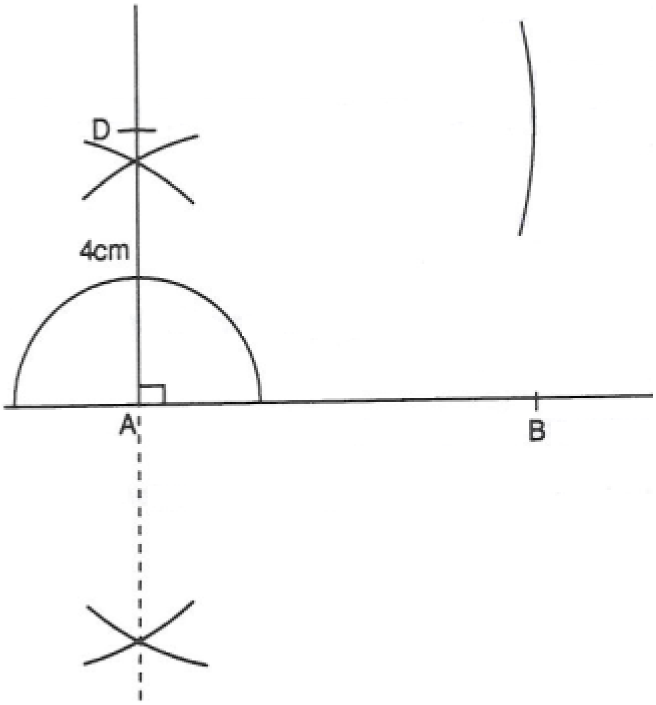
b) Construct a perpendicular at A (See Construction 4).



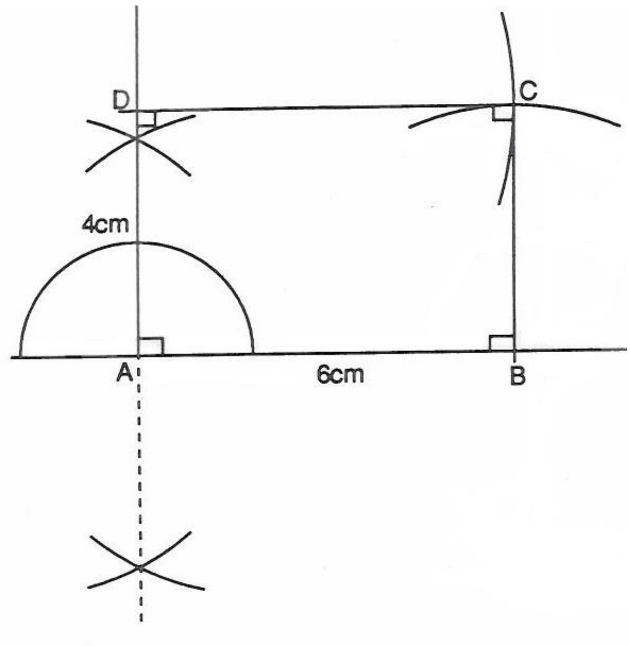
c) Open your compasses to 4 cm. Put the point on A, and draw an arc, which cuts the perpendicular. This gives you point D.



d) Open your compasses to 6 cm, and place the point on D, and make an arc, above line AB.



e) Open your compasses to 4 cm, and put the point on B, and make an arc. This should cross the arc from D, and will form point C.



f) Join point B to C, and point D to C. This forms the rectangle which measures 4 cm x 6 cm.

SCALE DRAWINGS.

When plans of houses, new roads etc are drawn up, they are usually drawn to “a scale”.

E.g. 1 cm = 10 km.
1 cm = 1 km.

This means that 1 cm on the drawing represents 10 m in “real life”. 1 cm on the drawing represents 1 km in “real life”.

On maps, scales are often given as a ratio.

E.g. 1 : 1000 See pack on Ratio and Proportion.

Now read through the following example:

Example

Queens Way is an interesting new development specially designed to meet the requirements of the first-time buyers. The houses are built of red bricks with a concrete tiled roof and offer the following accommodation: -

GROUND FLOOR

Covered Entrance Porch
Well-proportioned Dining Kitchen, 5 m x 3 m, with stainless steel sink unit, range of fitted base units and heat resistant working surfaces, provision for gas or electric cooker, wall mounted gas fired central heating boiler.

Sitting Room, 5 m x 4 m, with door to rear garden.

FIRST FLOOR

Bedroom, 5 m x 2.5 m

Bedroom, 4 m x 2.5 m

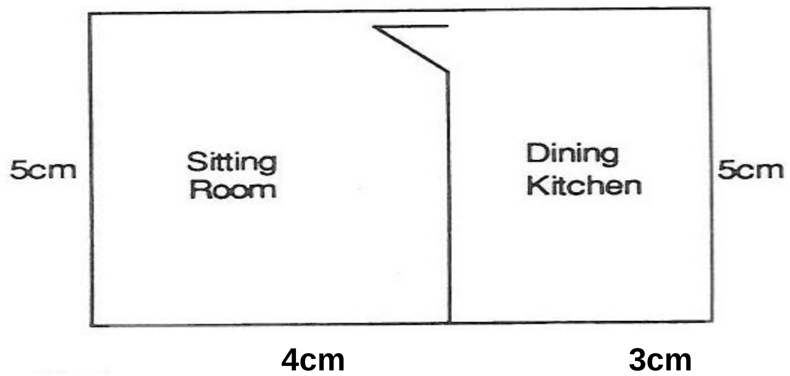
Bathroom with 3-piece suite comprising panel bath, pedestal toilet basin and low-suite WC.

OUTSIDE

2 Private Parking Spaces to each property.

If the ground floor is rectangular. Draw a scaled diagram of the ground floor.

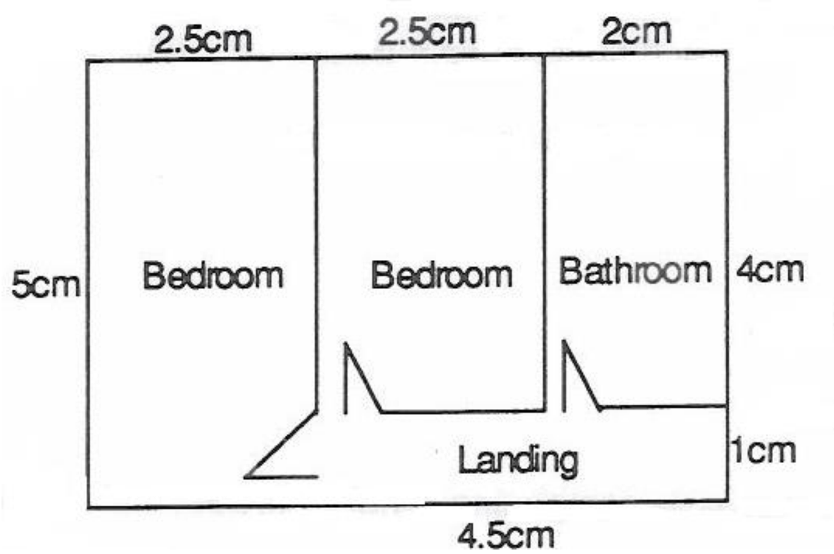
Draw a scaled diagram of a possible arrangement for upstairs.



Decide upon a scale, e.g. 1cm:1m.

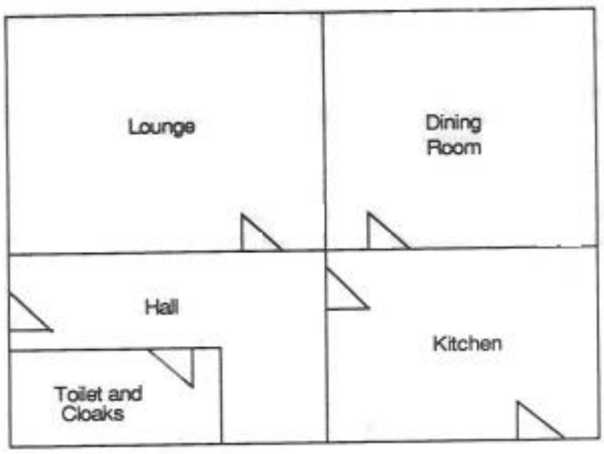
FIRST FLOOR

Bedroom, 5 m x 2.5 m
Bedroom, 4 m x 2.5 m
Bathroom, 4 m x 2 m
Landing, 1 m x 4.5 m



Exercise 1

1. The diagram below is the plan of the ground floor of a house.



Scale 1cm =1m

What is the area of the:

i) dining room
ii) lounge
iii) kitchen

2. Using a suitable scale of your choice, draw a scaled diagram of the front of the following house. The front view of the house is like a rectangle with a trapezium shaped roof on top. This roof is symmetrical about the centre and the sloping sides (8 m long) are at an angle of 45^0 to the horizontal. The width of the house is 22 m and the height to the start of the roof is 18 m.

   From your drawing, find

   i) the overall height of the house
   ii) the actual length of the horizontal part of the top of the roof.

LOCI.

The LOCUS of a point is simply the **path** it traces out if it moves according to some fixed **rule**.

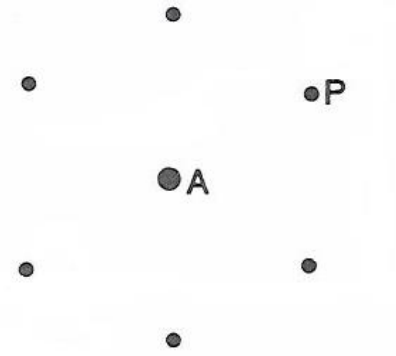
You can often see what this path will be by making some possible positions and "joining the dots"!

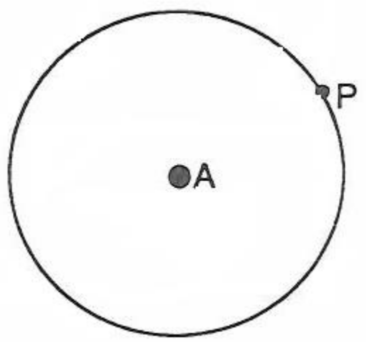
Example 1

Find the locus of a point P which moves so that it is always 2 cm away from a fixed-point A.

Solution

1. Mark the point A.
2. Dot in some possible positions for P.
3. By joining up **all** possible positions, we would obtain a circle A radius 2 cm.





Example 2

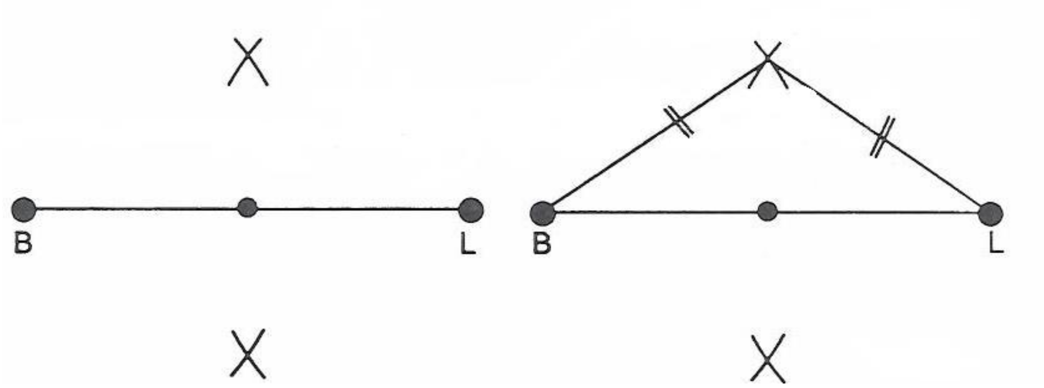
Find the locus of a point P such that it is equidistant from 2 fixed points B and L.

Solution

1. Mark the points B and L.

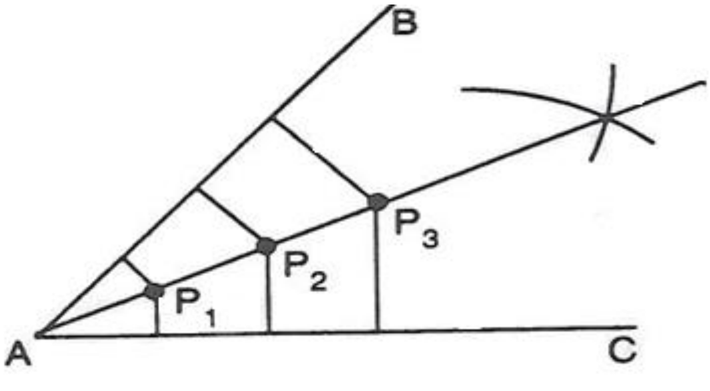
2. Suppose B and L represent Bradford and Leeds on a map. Where could you possibly live to be at equal distances from the two towns?
One obvious position is exactly midway between them. But are there any others?

3. Mark in positions to the North of BL and South of BL. By "joining the dots", we see we have the bisection of BL (Construction 2(i)).



Example 3

Find the locus of a point P which is equidistant from 2 intersecting lines, AB and AC.



We require possible positions to be the same perpendicular distance from the line. Put in some possible positions P_1, P_2 , P_3. We have the bisector of angle BAC. (Construction 3)

Exercise 2

1. A holiday caravan site is bounded by three straight fences AB = 140 m, BC = 120 m and AC = 75 m. A telephone kiosk, T, on the site is equidistant from AB and BC and also 90 m from B. Using a ruler and compasses only, show a scaled diagram the relative positions of A, B and T. Measure and state the distance CT.

2. Construct the triangle ABC, where AB = 9 cm, BC = 11 cm and angle ABC = 120^0.

   On you figure, construct:

   i) The perpendicular from B to AC,
   ii) The bisector of the angle BAC.

3. Use only ruler and compasses in this question, and show clearly all your construct arcs:

   a) Draw a horizontal line PQ = 15 cm, construct the quadrilateral PQRS with PS = 9.1 cm,
   b) SR = 9.4 cm, RQ = 10.7 cm, and angle QPS = 60^0.
   c) Construct the bisector of the angle PQR,
   d) Construct the perpendicular from R to this bisector.

4. a) Construct a triangle ABD, such that AB = 9.2 cm, AD = 7.4 cm, and the angle DAB = 60^0.

   b) Using a ruler and a protractor, complete the quadrilateral ABCD such that the angle DBC = 35^0 and BC = 8.5 cm.

   c) Using a ruler and a protractor, draw another triangle, BCE such that angle BCE = 35^0 and E is a point on the line AB produce

   d) Using a ruler and compasses only, construct the perpendicular from B to CE.

5. Draw a horizontal line AB = 15 cm long. Using this line, construct:

   a) the angle ABC = 60^0 at the point B.

   b) the quadrilateral ABCD such that AB = 15 cm, AD = 10.2 cm, CD = 9.4 cm and BC = 8.6 cm, and angle ABC = 60^0

c) on your figure, construct:-

   i) the locus of point which are equidistant from the points C and D,
   ii) the locus of point which are equidistant from the line AB and AD, and which lie within the quadrilateral ABCD.

d) Mark clearly on your figure a point P, which is both equidistant from C and D, and equidistant from AB and AD.

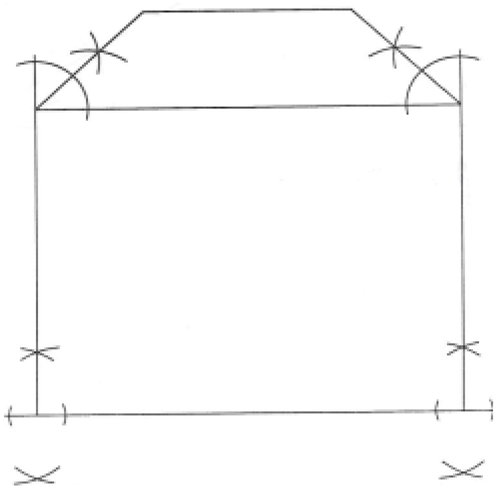
ANSWERS

Exercise 1

1. i) $5 \times 5 = 25\ m^2$
   ii) $6 \times 5 = 30\ m^2$
   iii) $4 \times 5 = 20\ m^2$

   First always draw a rough diagram of the information you are given.

2.

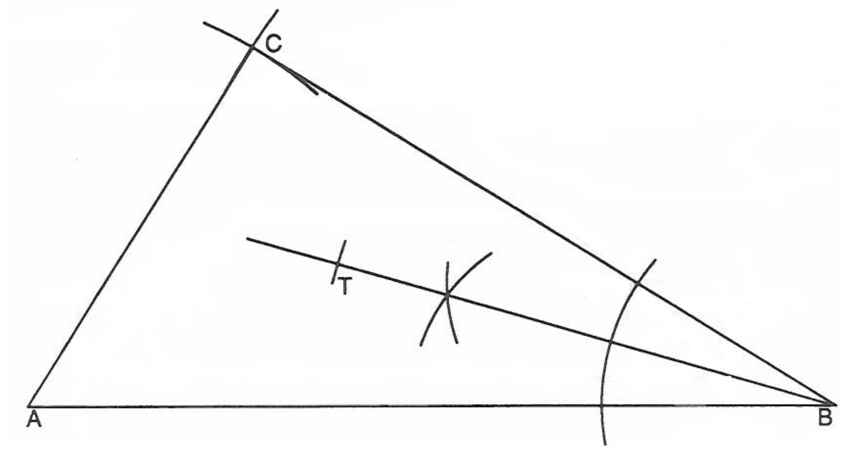


Scale Let 1 cm = 2 m

i) 23.5 m
ii) 11m

Exercise 2

1.

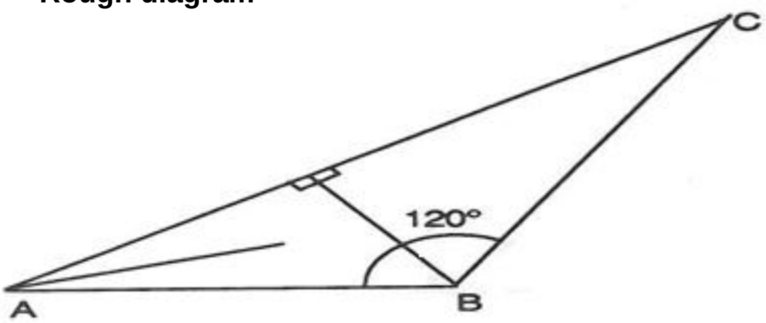


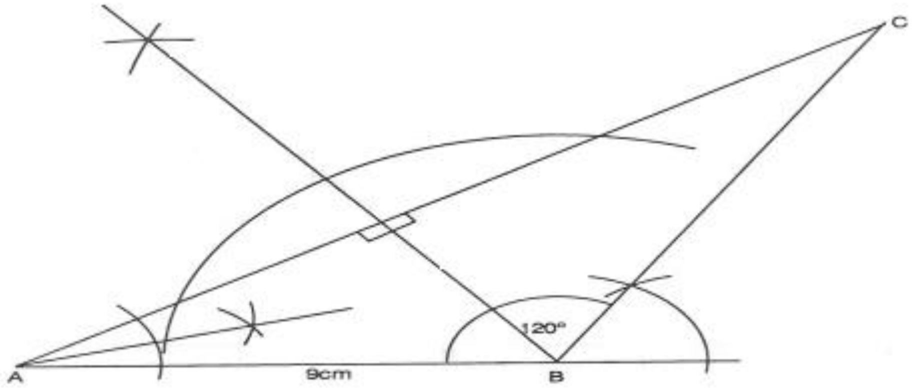
Scale 1 cm = 10 m.

CT = 4.2 cm.
CT = 42 m.

2.

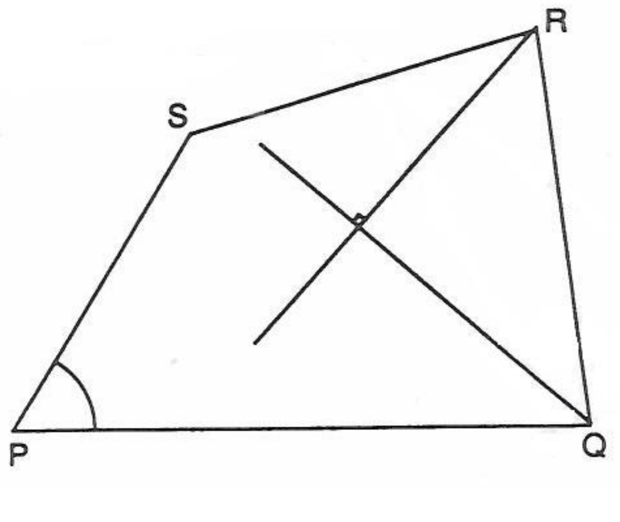
Rough diagram

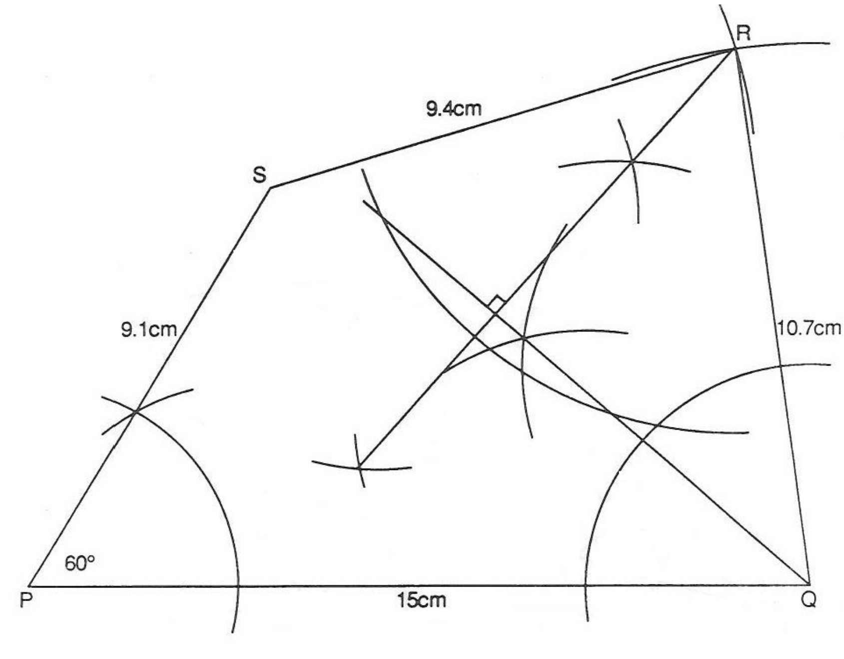




3. a) b) c)

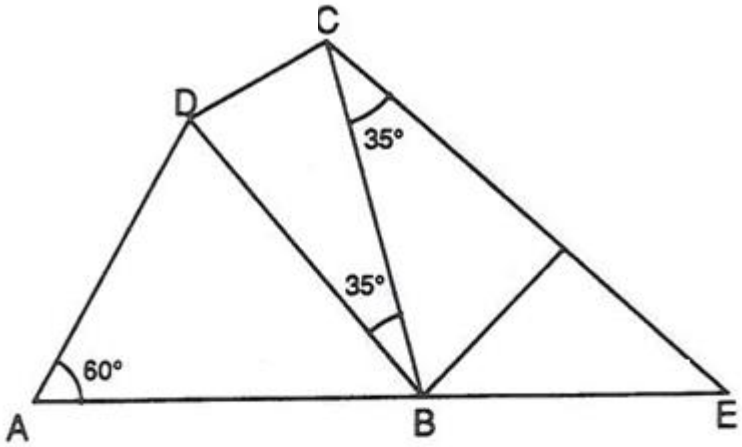
Rough diagram



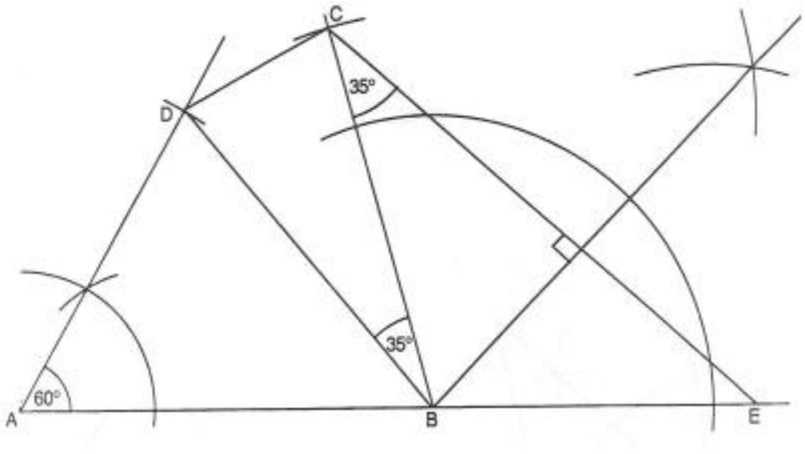


4.

Rough diagram

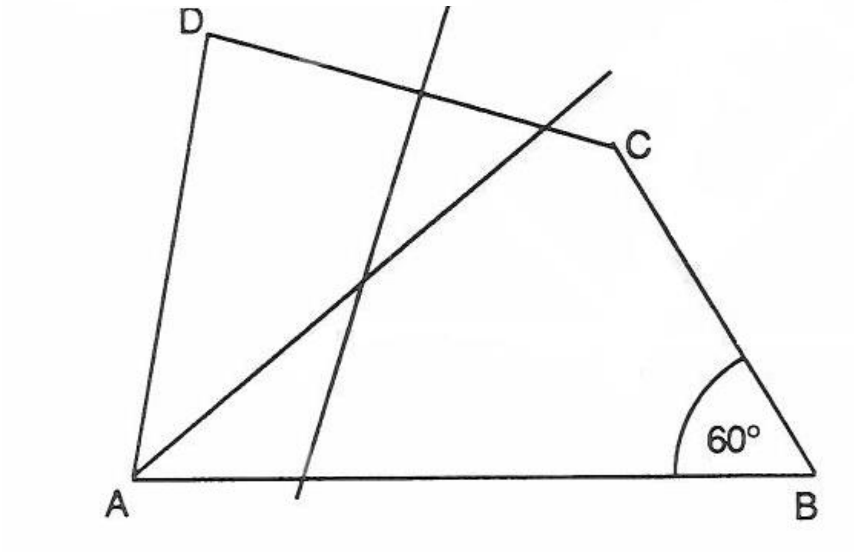


a) and b)
c) and d)



5.

Rough diagram



i), ii), iii), a, b, iv).