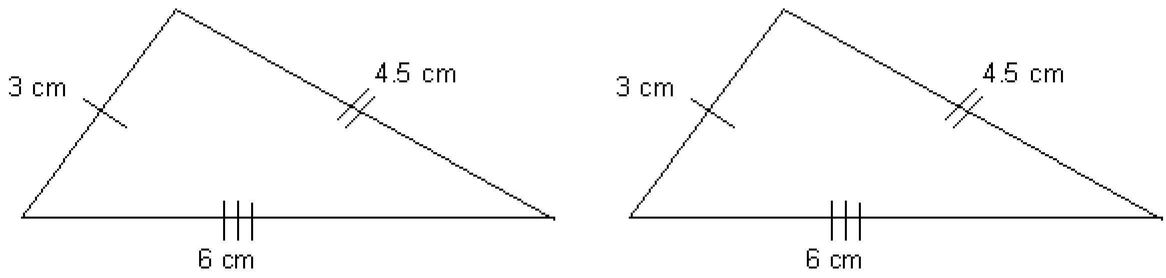
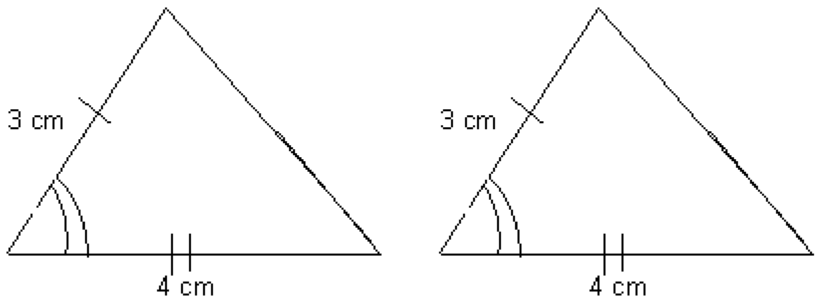
CONGRUENT TRIANGLES

Congruent triangles are triangles that are exactly alike in every aspect. There are **FOUR** sets of rules which can be applied to congruent triangles.

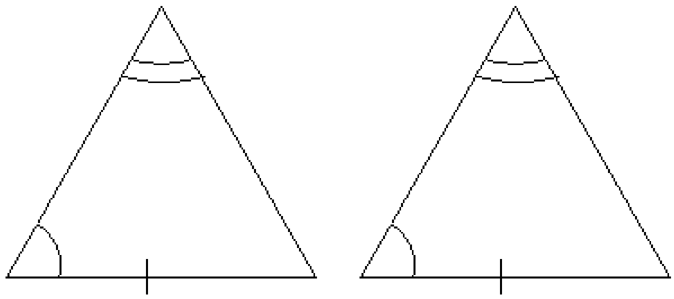
1. Triangles are congruent if 3 sides in one triangle are equal to the corresponding sides in another triangle.



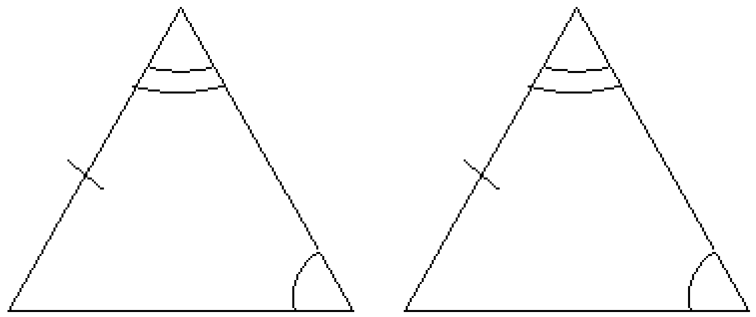
2. Triangles are congruent if two sides and the **angle between them** in one triangle are equal to two sides and **angle between them** the in the other triangle.



3. Triangles are congruent if one side and two angles in one triangle are equal to one side and two angles in the same position in the other direction.



OR



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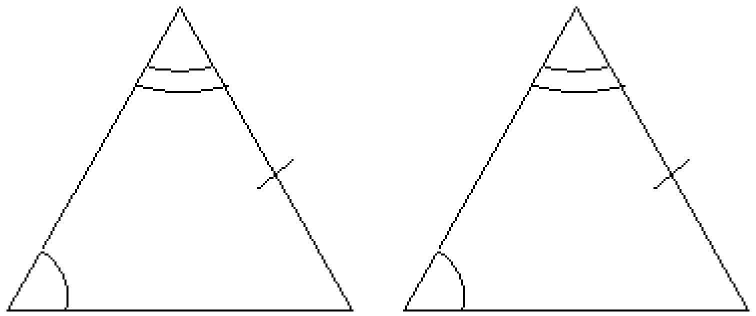

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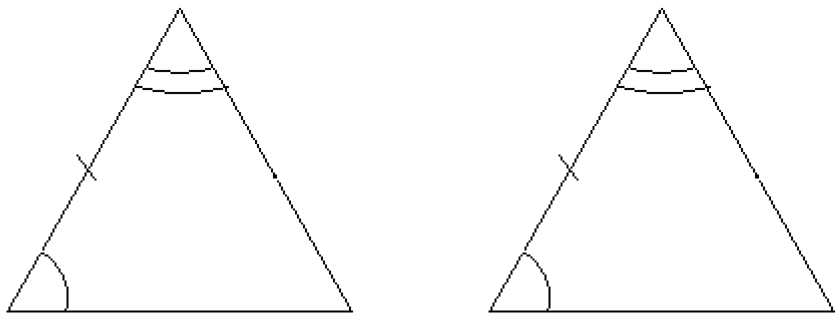
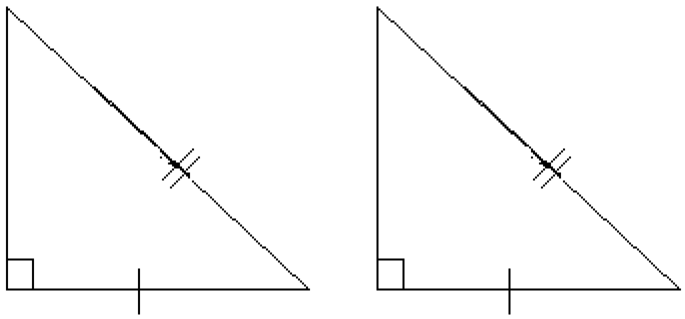


AAS
(Angle - Angle- Corresponding Side)

4. Triangles are congruent if they are right-angled and the hypotenuse in one equals the hypotenuse in the other, and one side equals the corresponding side in the other.

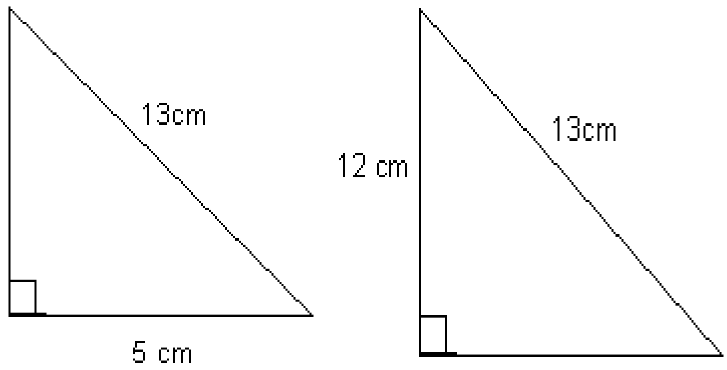


RHS
(**Right-Angle – Hypotenuse - Corresponding Side**)

Exercise 1

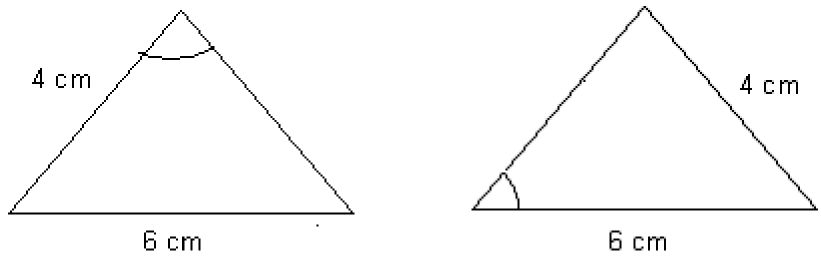
Look at these and identify which are congruent pairs. Answer yes or no.

1.



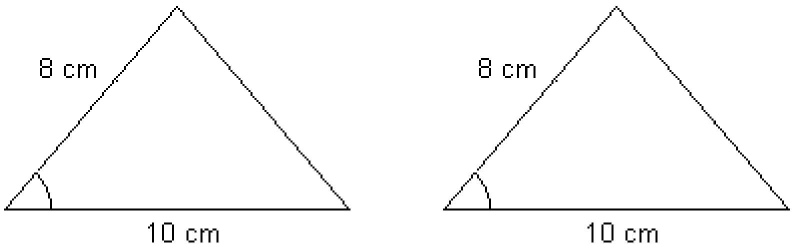
YES or NO

2.



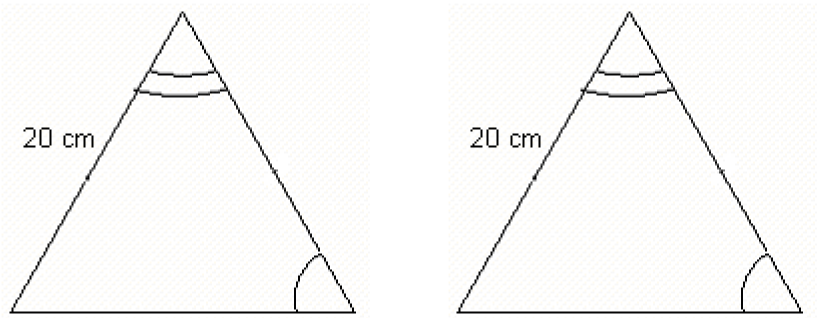
YES or NO

3.



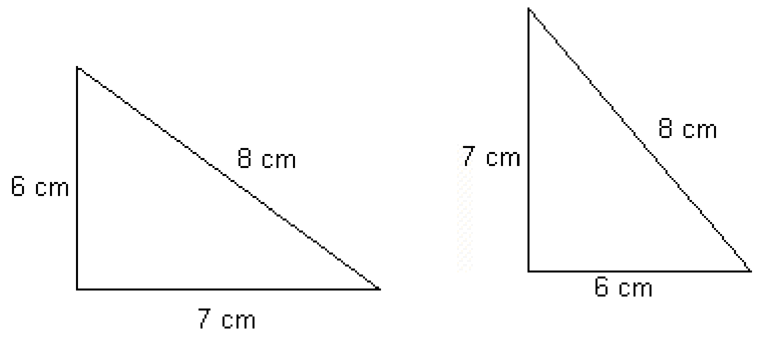
YES or NO

4.



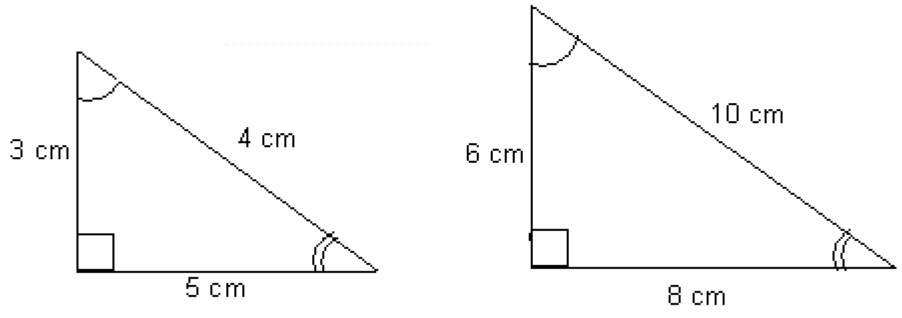
YES or NO

5.



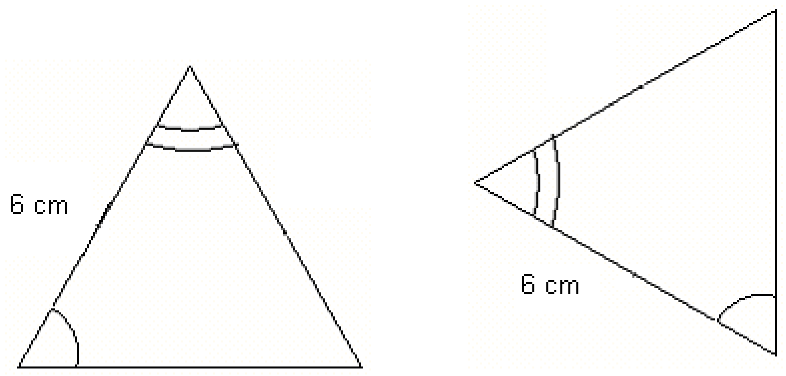
YES or NO

6.



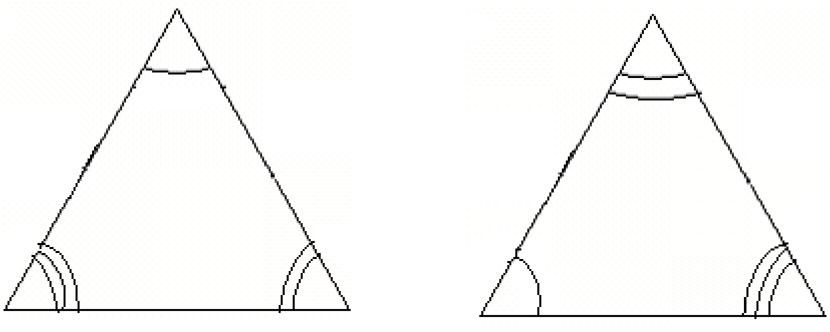
YES or NO

7.



YES or NO

8.

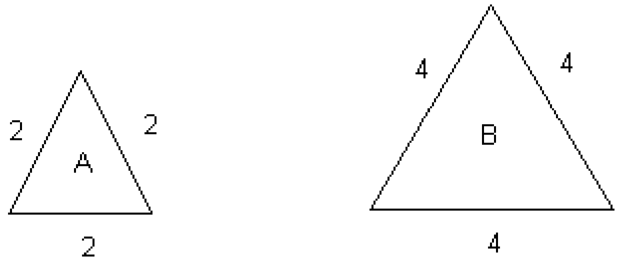


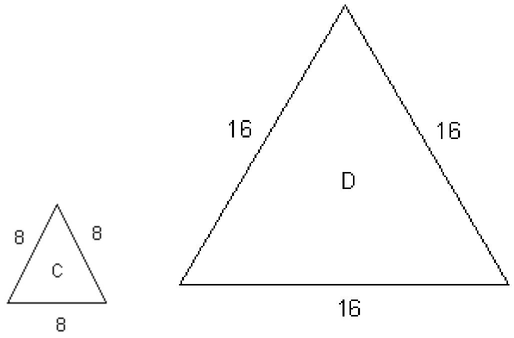
YES or NO

SIMILAR TRIANGLES

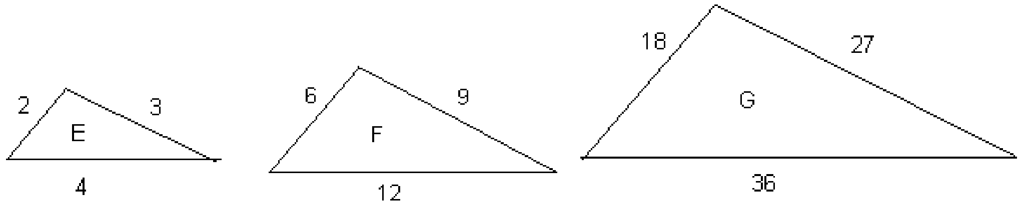
Why were number 6 and number 8 in the pervious example **NOT** congruent?

You will notice that the angles are equal, but in 6 the measurements of the sides are not equal, they are twice as big. We can draw many different sized triangles **with the same angles**, but having different sizes.
The important point to remember in similar triangles is that the angles are the same and the corresponding sides are in the same ratio.





These triangles are similar. The sides are in the ratio 2 : 4 or 1 : 2.
A : B = 1 : 2
C : D = 1 : 2



E : F = 2 : 6 or 4 : 12 or 3 : 9

E : F = 1 : 3 1 : 3 1 : 3

F : G = 6 : 18 or 12 : 36 or 9 : 27

F : G = 1 : 3 1 : 3 1 : 3

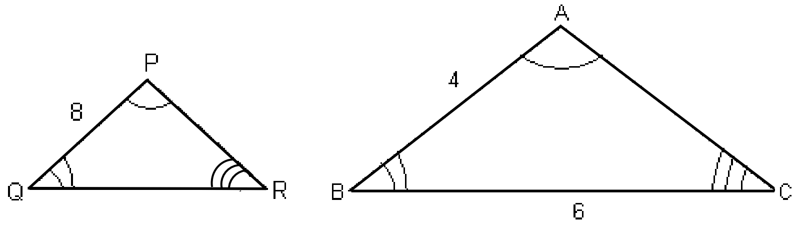
NB It does not matter which pair of sides you compare so long as they are **CORRESPONDING** sides.

Calculating Lengths of Sides of Similar Triangles

In this type of question, you will be asked to find the missing side. So first establish whether or not they are similar.

Example 1

Find QR



Triangle **ABC** and triangle **PQR** are similar, compare corresponding sides.

Sides which correspond are:	A and PQ	A = P
	BC and QR	B = Q
	AC and PR	C = R

We have a measurement for AB and BC; so we can use an equation to find QR.

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

Now substitute the values you are given

$$\frac{4}{8} = \frac{6}{QR}$$

Now cross multiply to give

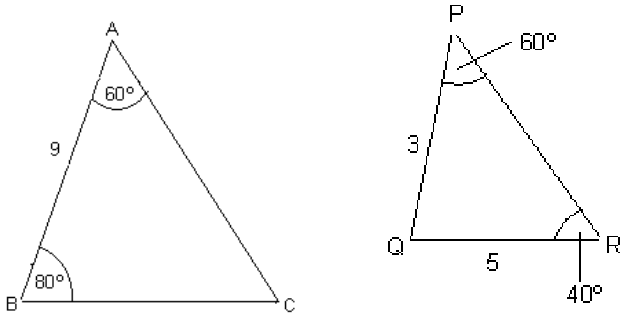
$$4QR = 6 \text{ X } 8 = 48$$

$$QR = \frac{48}{4}$$

$$QR = 12$$

Example 2

Find BC



Are these triangles **SIMILAR?**

In triangle ABC we have : C = 40 (Angles in a triangle equal 180º)
And in triangle PQR we have: Q = 80 (Angles in a triangle equal 180º)
Hence: A = P
B = Q
C = R

Thus, triangles ABC are similar
PQR

[**NOTE:** Write the equal angles underneath each other.]

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

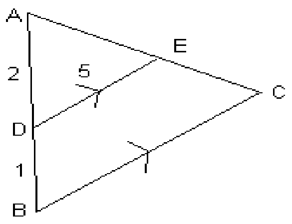
or

$$\frac{9}{3} = \frac{BC}{5}$$

Whence BC $= \frac{9 \times 5}{3} = \underline{\mathbf{15}}$

Example 3

Find BC



In triangles ADE and ABC we have:
A = A (common)
D = B (corresponding angles)
E = C (corresponding angles)

Thus, triangles ADE are similar
ABC

$$\frac{AD}{AB} = \frac{DE}{BC}$$ (**NB** AB = 2+ 1 = 3)

or

$$\frac{2}{3} = \frac{5}{BC}$$

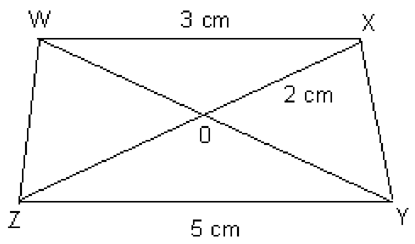
Whence 2BC = 15

$$BC = \frac{15}{2}$$

= **7.5**

Example 4

Find OZ



[**NOTE:** the parallel lines]

In triangles WOX and ZOY we have: O = O (vert. opp. angles)
W = Y (alt. angles)
X = Z (third angles of triangles)

Hence, triangles OWX are similar to OYZ.

$$\frac{OX}{OZ} = \frac{WX}{YX}$$

$$\frac{2}{OZ} = \frac{3}{5}$$

or

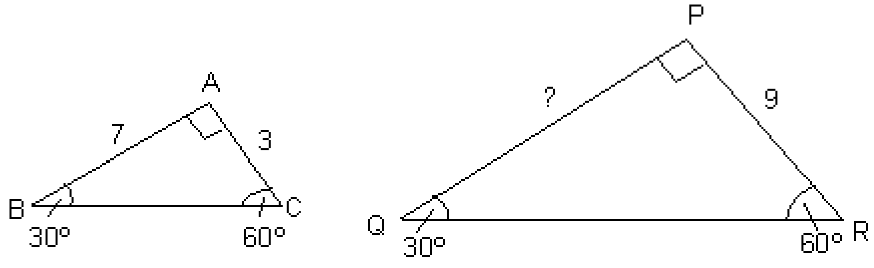
$$3OZ = 10$$

$$OZ = \frac{10}{3} = \mathbf{3} \ \frac{1}{3} \text{cm}$$

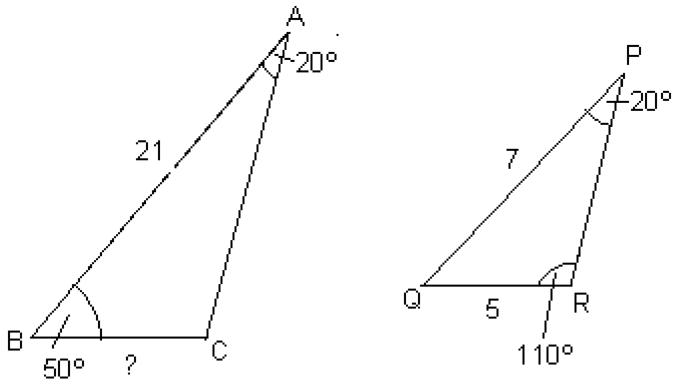
Exercise 2

First identify triangles as similar. Then find the missing measurement as before.

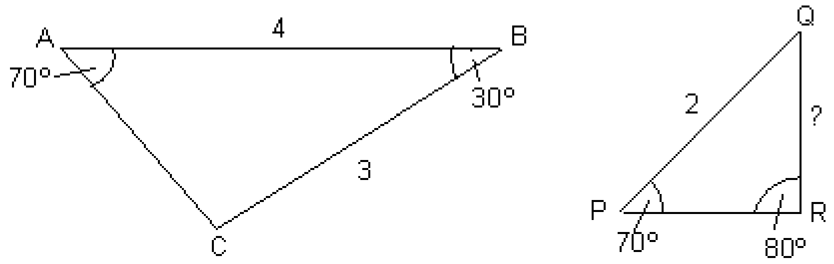
1. Find PQ



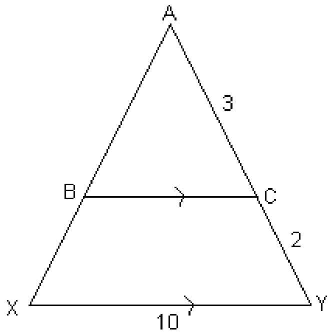
2. Find BC



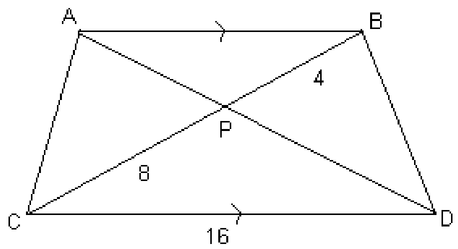
3. Find QR



4. Find BC



5. Find AB



Areas of similar triangles

As already stated, the ratio of the areas of similar shapes is equal to the ratio of the squares of the corresponding sides.

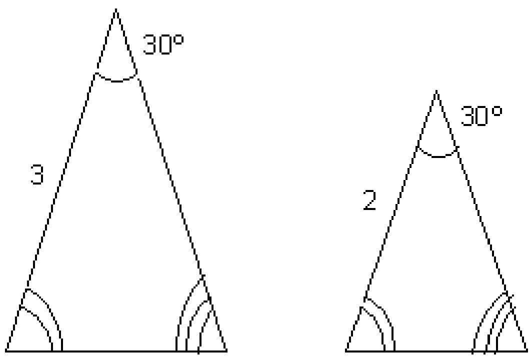
So if you have 2 sides in the ratio 1:3. The ratio of the areas will be

$1^2 : 2^2$

$1 : 9$

<u>Sides 2 : 3</u>	**<u>Sides 3 : 10</u>**
Areas $2^2 : 3^2$ $4 : 9$	Areas $3^2 : 10^2$ $9 : 100$

Example 1

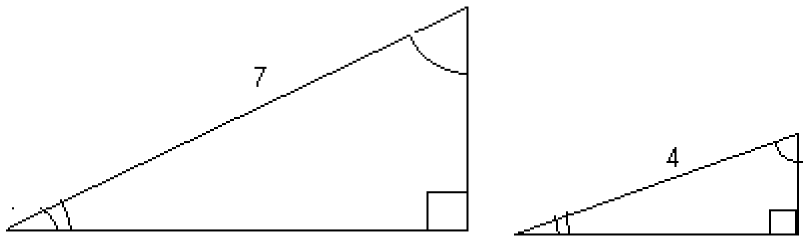


Ratio of sides is $3 : 2$

Ratio of areas is $3^2 : 2^2$

$= 9 : 4$

Example 2

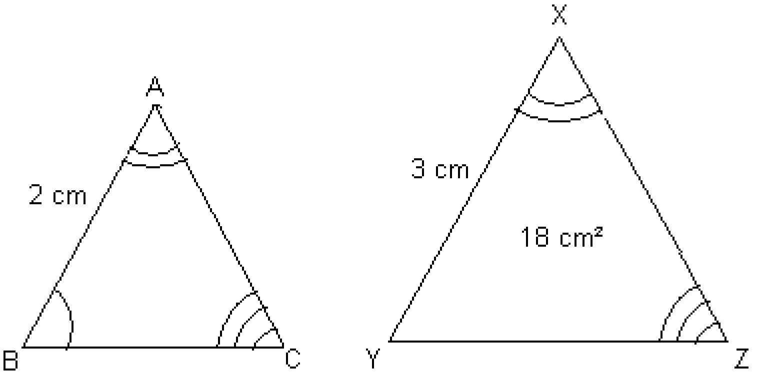


Ratio of sides is $7 : 4$

Ratio of areas is $7^2 : 4^2$

$= 49 : 16$

Example 3



The triangles are similar (3 angles same). You are asked to find the area of triangle ABC. First- compare corresponding sides.

AB : XY

Substitute 2 : 3

Ratio of area $2^2 : 3^2$

4:9

$$\frac{4}{9} = \frac{\text{Area of trianlge ABC}}{18\,(\text{Area of trianlge XYZ})}$$

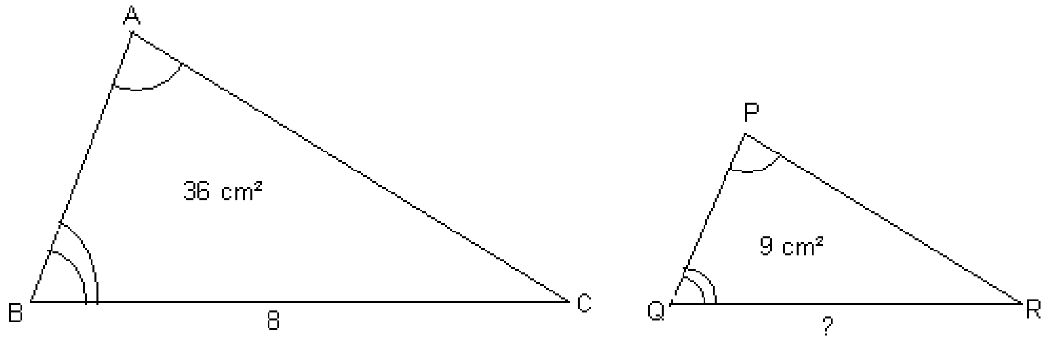
Cross multiply 18 x 4 = Area of ABC x 9

$$\frac{18 \times 4}{9} = \text{Area of ABC}$$

8 = Area of ABC

As you know area is measured in square units so the area = 8 cm².

Example 4



The triangles are similar:-

Find QR

Ratio of sides 8 : QR
Ratio of areas $8^2 : QR^2$
$64 : QR^2$

$$\frac{64}{QR^2} = \frac{36}{9}$$

Cross multiply $9 \times 64 = 36 \times QR^2$

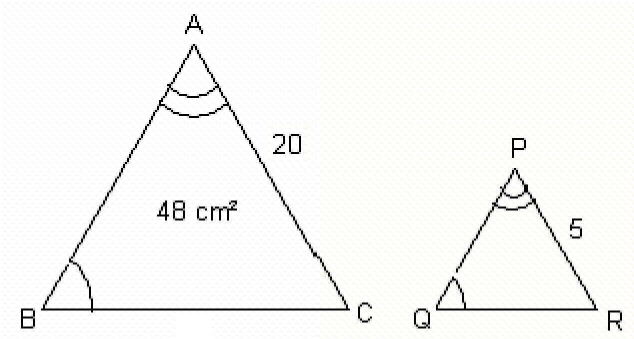
$$\frac{9 \times 64}{36} = QR^2$$

$16 = QR^2$

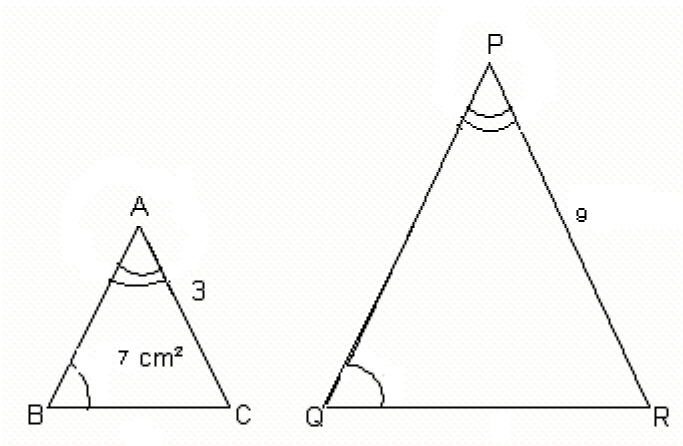
$\sqrt{16} = QR = 4$
QR = 4

Exercise 3

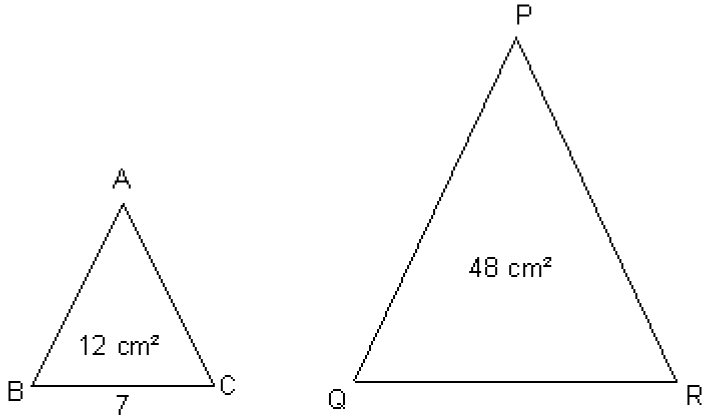
1. Find area of PQR



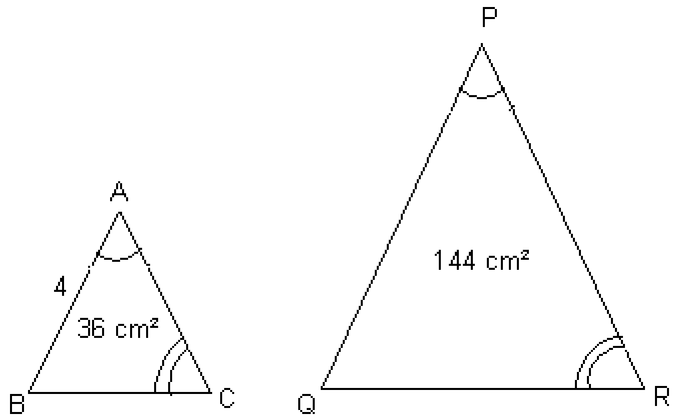
2. Find area of PQR



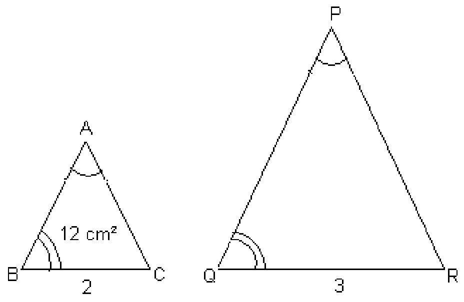
3. Find area of QR



4. Find area of PQ



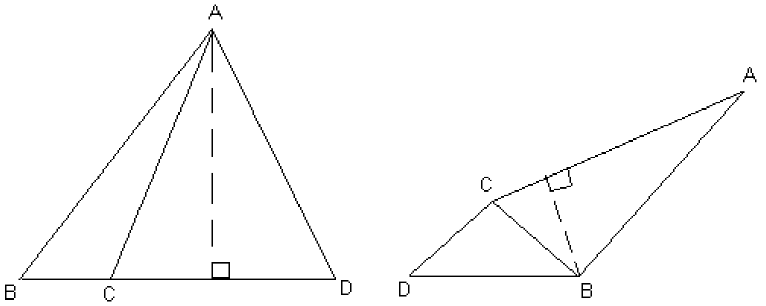
5. Find area of PQ



Comparing Areas of Triangles

You have been shown one way of comparing areas, using similar triangles.
Another method, which we shall call the **same height method**, is now given.
If the triangles **are not similar** but **have the same height** the following method must be used.

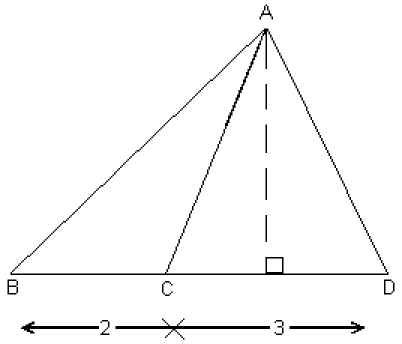
NOTE- You will **NOT** have to work out the actual values of sides and area – your answers will always be in the form of a ratio. Take care when labelling the known sides.



Triangle ABC and triangle ABD have the same height.

Example 1

Compare the areas of triangle ABC and ACD in the diagram below:



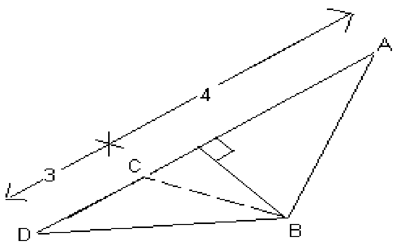
$$\frac{\text{Area of ABC}}{\text{Area of ACD}} = \frac{0.5 \text{ base x perp. height}}{0.5 \text{ base x perp. height}}$$

$$= \frac{2}{3}$$

Answer: **2 : 3**

Example 2

Compare triangle ABC and triangle ABD.



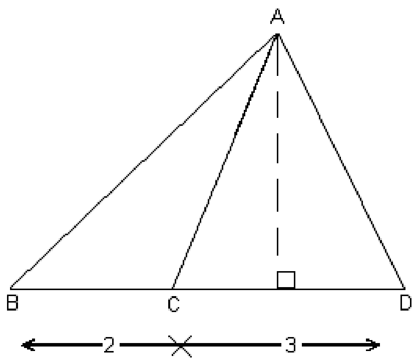
$$\frac{\text{Area of ABC}}{\text{Area of ABD}} = \frac{\text{0.5 base x perp. height}}{\text{0.5 base x perp. height}}$$

$$= \frac{4}{7} \quad \text{(base 4 + 3 = 7)}$$

Answer: **4 : 7**

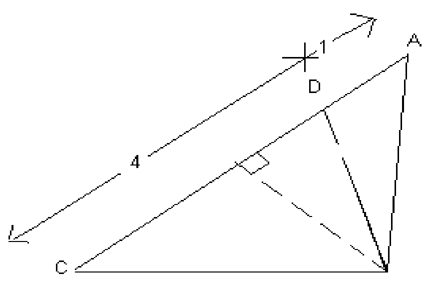
Exercise 4

1.



$$\frac{\text{Triangle ABC}}{\text{Triangle ABD}^{2}}$$

2.



$$\frac{\text{Triangle ABD}}{\text{Triangle BCD}}$$

Learning Development

University of Northampton

ANSWERS

Exercise 1

1.	Yes	2.	No	3.	Yes	4.	Yes
5.	Yes	6.	No	7.	Yes	8.	No

Exercise 2

1.	PQ = 21	2.	BC = 15	3.	QR = 1.5	4.	BC = 6 cm
5.	AB = 8						

Exercise 3

1.	PQR = 3cm^2	2.	PQR = 63 cm^2
3.	QR = 14 cm	4.	PQ = 8cm
5.	PQR = 27 cm^2		

Exercise 4

1. $$\frac{\text{Area of ABC}}{\text{Area of ABD}} = \frac{2}{5}$$

2. $$\frac{\text{Area of ADB}}{\text{Area of BCD}} = \frac{1}{4}$$