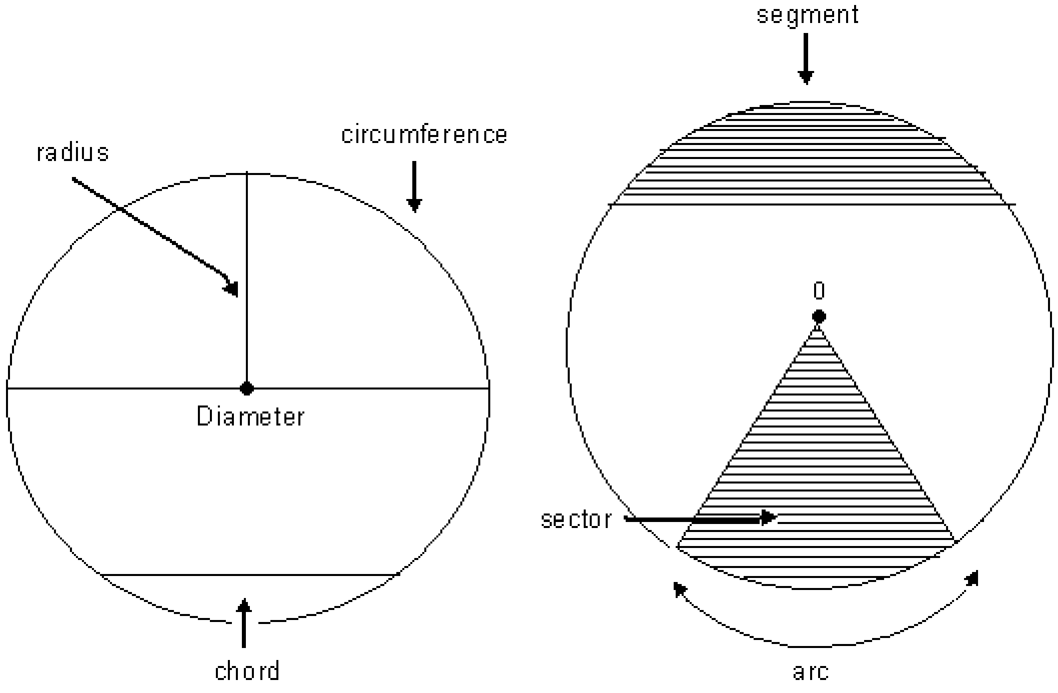
Learning Development

University of Northampton

CIRCLES

Look at these diagrams, then at the list below.



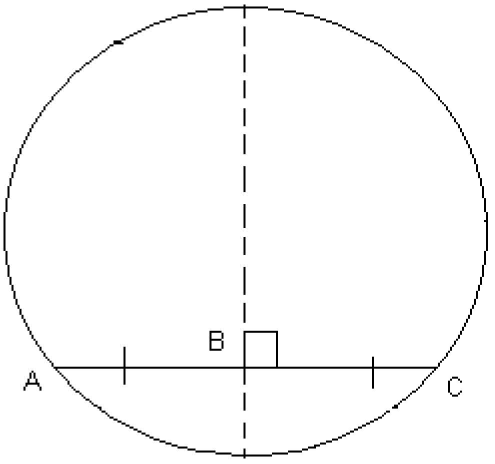
Words you will use and explanations of these words:

Arc - an Arc is a part of the circumference.
Centre - the Centre of a circle is usually marked 0.
Chord - a Chord is a line drawn from one side of the circumference to the other.
Circle Circumference
- the Circumference is the “perimeter of a circle (that is, the distance round the circle)
Diameter - when a Chord is drawn from one side of the circumference to the other, passing through the centre. This is a special chord.
Quadrant - a Quadrant is a quarter of a circle
Radius or more than one radius = radii
- a Radius is a line drawn from the centre to the circumference.
Sector - a Sector is a part of the circle bounded by the 2 radii and an arc.
Segment - a Segment is a part of the circle bounded by a chord and an arc.
Semi-circle - is half a circle.

Here are the explanations of these words.

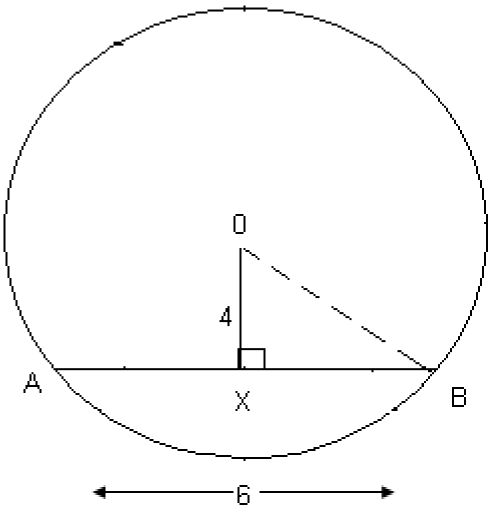
There are certain rules applying to circles, which must be learnt. Please **read** and make notes of the following.

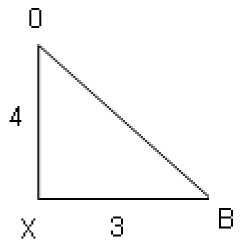
1. If a diameter of a circle is at right-angles, to a chord then it cuts the chord, then it cuts into equal parts.
   i.e. AB = BC



Example 1

Find the radius





OX cuts AB in half

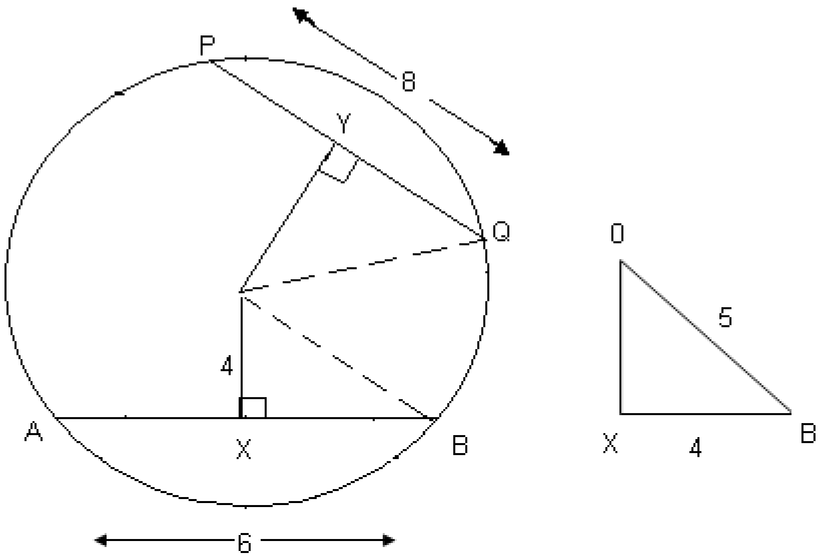
AX = 3
BX= 3

OXB is right angled triangle. Use Pythagoras' Theorem to find OB

$OB^2 = OX^2 + BX^2$
$OB^2 = 4^2 + 3^2$
$OB^2 = 16 + 9 = 25$
$OB = 5$

Example 2

Find OY



First, **draw in the radius as shown**. As in example one, use Pythagoras' Theorem to find OB which is 5 (again 5!)

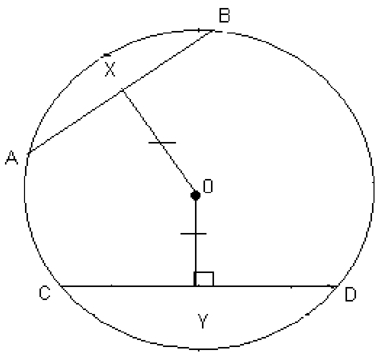
0Y cuts PQ in half
PY= 4cm and QY = 4cm
0Q is a radius
0Q = 0B = 5cm

Find 0Y by using Pythagoras' Theorem

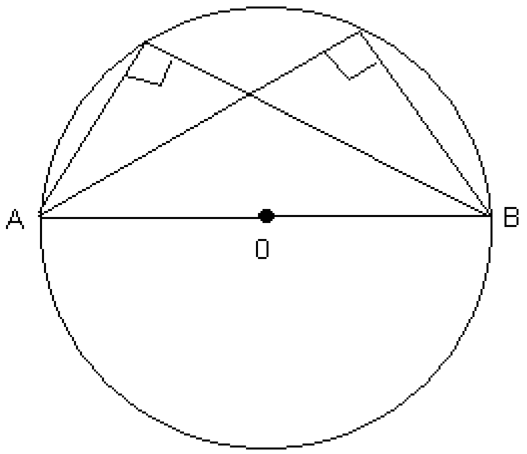
$0Q^2 = 0Y^2 + YQ^2$
$0Y^2 = 0Q^2 - YQ^2$
$= 25 - 16 = 9$
$0Y = 3$

2. Chords which are equal in length are the same distance from the centre of the circle.

i.e. If AB = CD
then 0X = 0Y

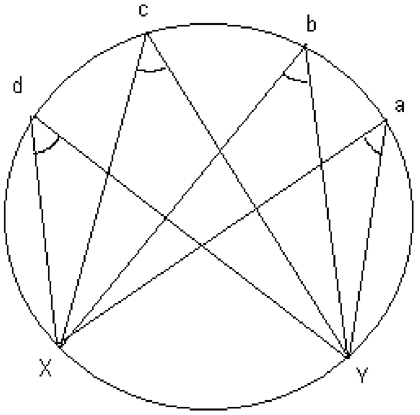


3. AB is a diameter. The angle in a semi-circle is a right-angle.

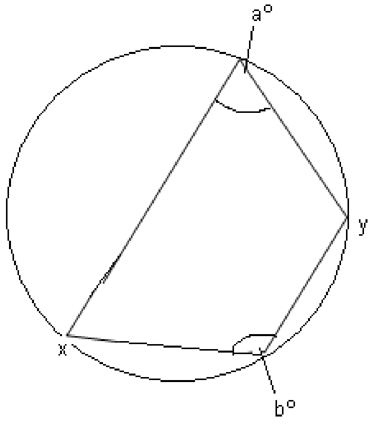


4. Angles subtended at the circumference by the same arc in the same segment of the circle are equal.

$a^o = b^o = c^o = d^o$

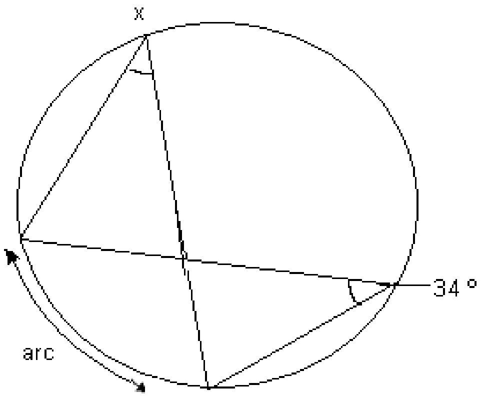


NOTE: in the diagram below, a^o **does not equal** b^o because a is pointing "upwards" and b is pointing "downwards".



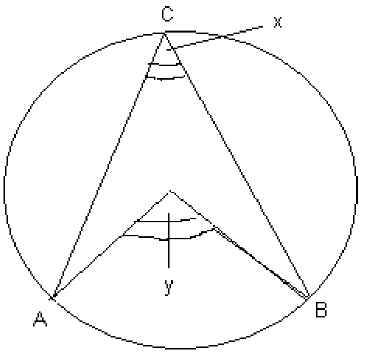
Example

Find x



$x = 34^o$ (angle subtended at the circumference by the same arc)

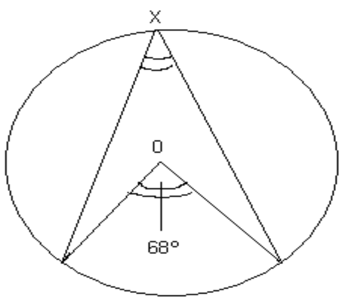
5. 0 is the centre. The angle subtended at the centre is **twice** the angle subtended at the circumference **if not on the same arc** and in the same part of the circle.



This means that $2x^o = y^o$

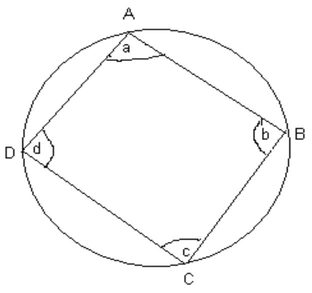
Example

Find x



$x = 34^o$ (Angle at the centre is twice the angle at the circumference.)

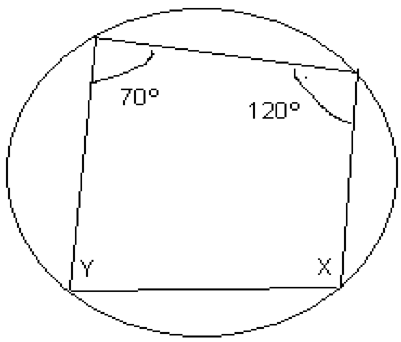
6. A four-sided figure inside a circle, as shown, with the four corners touching the circumference is called a **cyclic quadrilateral**.
Opposite angles of a cyclic quadrilateral add up to 180º.



Therefore,
Angles a + c = 180º
And b + d = 180º

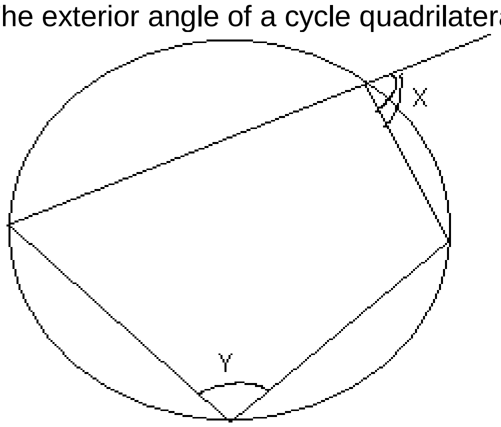
Example

Find *x* and y



x = 110º (180º - 70º)
y = 60º (180º - 120º)

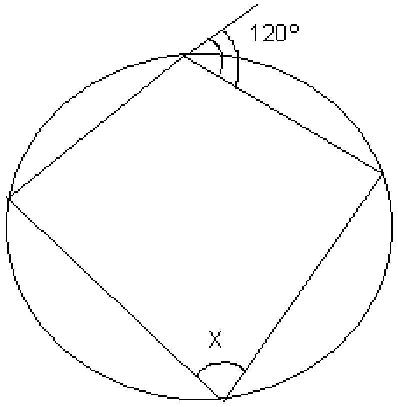
7. The exterior angle of a cycle quadrilateral is **equal to the opposite interior angle.**



i.e. *x* º = yº

Example 1

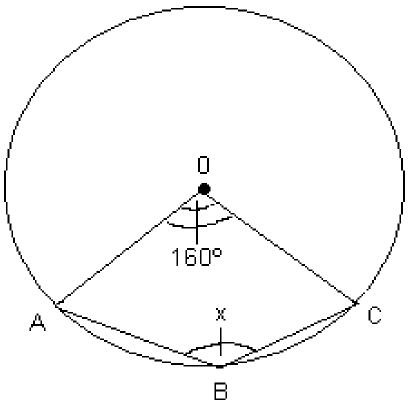
Find X



X = 120º

Example 2

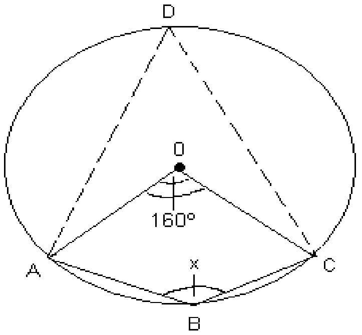
Find x



There are two possible methods of solving this:

a) A0CB is **not** a cyclic quadrilateral, because all four corners **do not touch the circumference**.

So, **draw a line**, as shown.



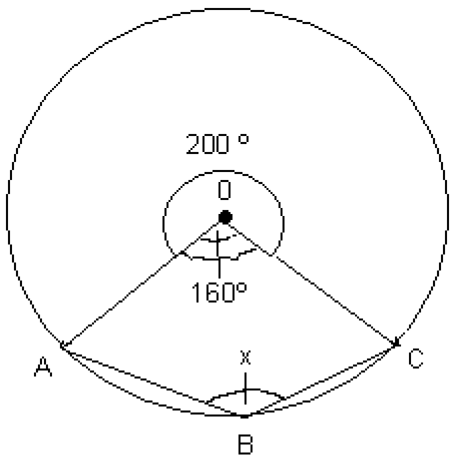
ABCD now a cyclic quadrilateral.

Now angle ADC = 80º (angle subtended at circumference is **half** that at the centre)

Angle ADC + Angle ABC = 180º. So, Angle ABC = 100º (180º - 80º)

OR

b) First find A0C, which is a **reflex angle.**



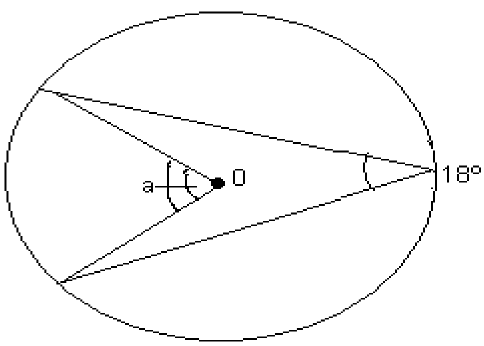
Angle A0C = 200º
(Angles round a point =360º)
So, Angle ABC = 100º
(Angle at the centre is **twice** the angle at the circumference).

Both methods are acceptable. Choose the one which you feel happier working with.

MORE EXAMPLES

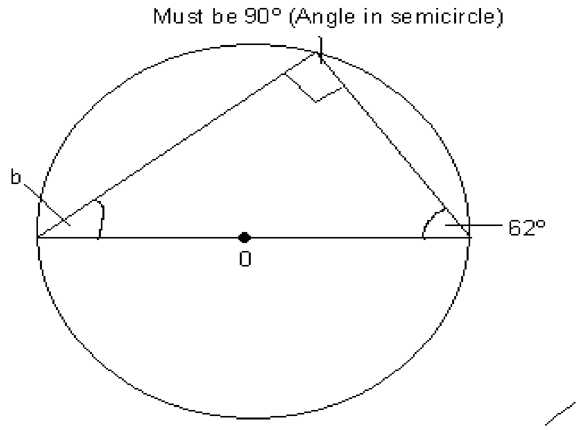
0 is the centre of the circle in all of the following examples.

i)



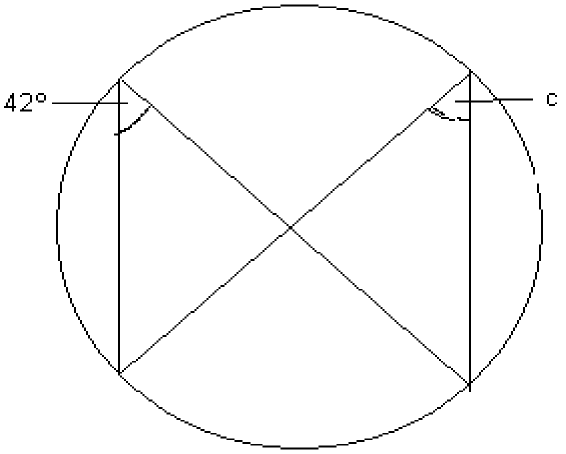
a= 36º (angle at centre is twice that at the circumference).

ii)



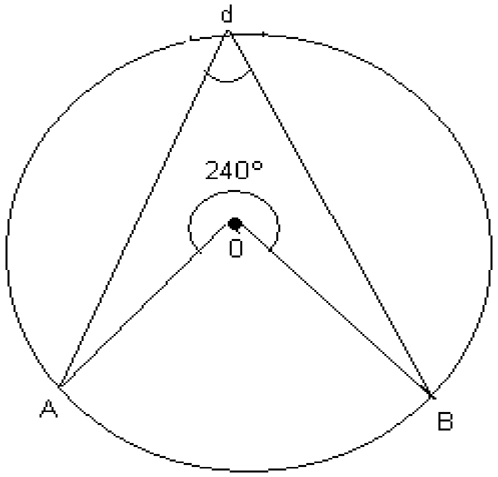
b= 90º – 62 = 28º (angle in a semicircle=90º)

iii)



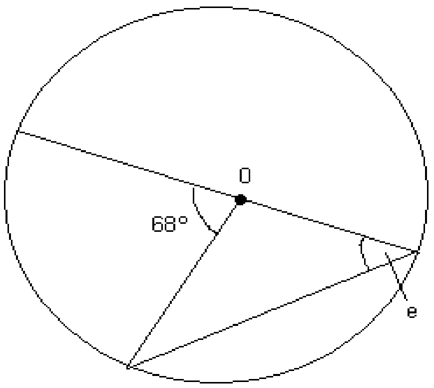
c= 42º (angles in same segment)

iv)



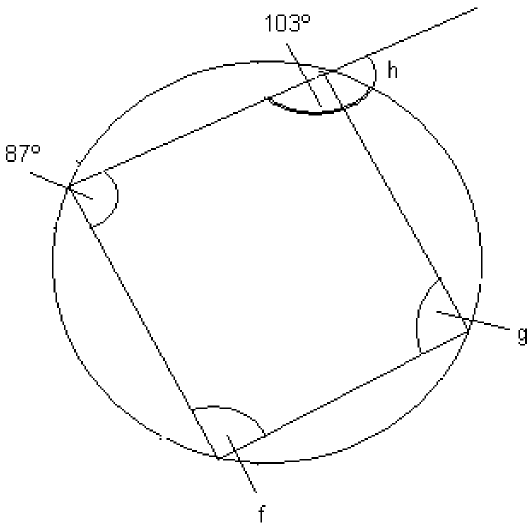
First find angle A0B. Angle A0B= 120º (angles around a point = 360º).
d = 60º (angle at centre is twice that of the circumference).

v)



e = 34º (Angle at centre = twice that at circumference)

vi)

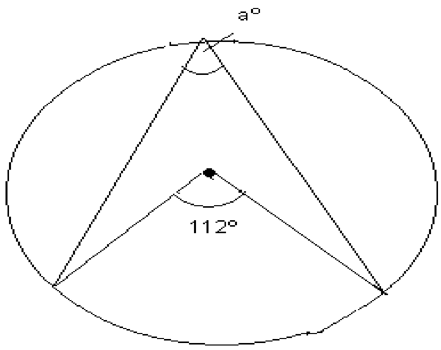


f = 77º (opposite angles in a cyclic quadrilateral)
g = 93º (same reason as above)
h = 77º (exterior angle of cyclic quadrilateral = opposite interior angle)

Exercise 1

0 is the centre of the circles in all cases.
Find the angles marked with the letters.

1.



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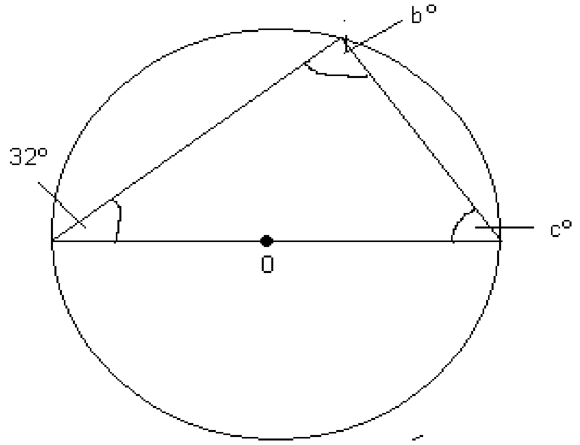

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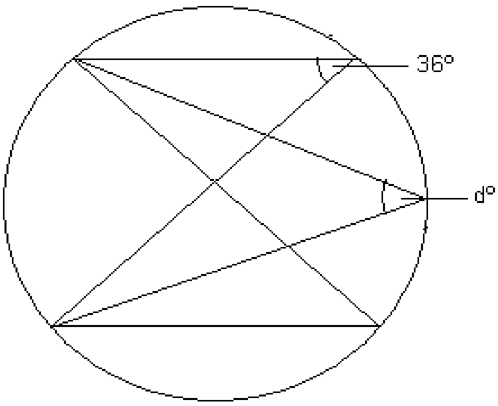
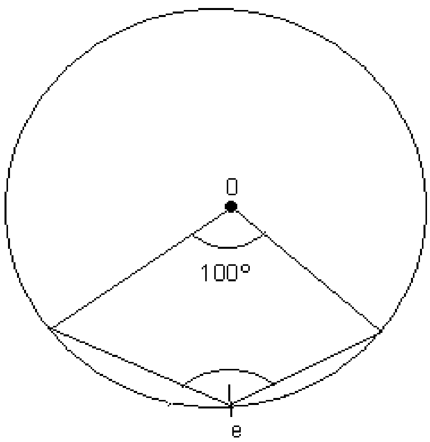
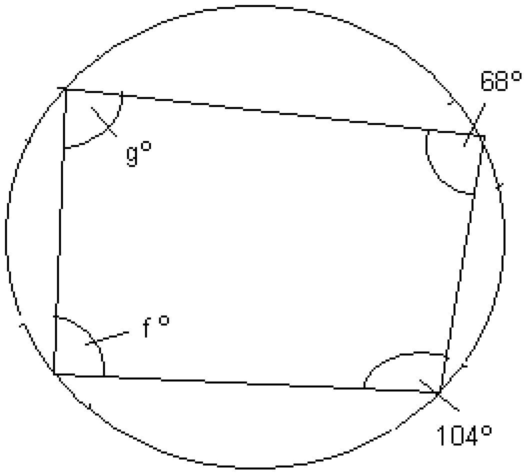


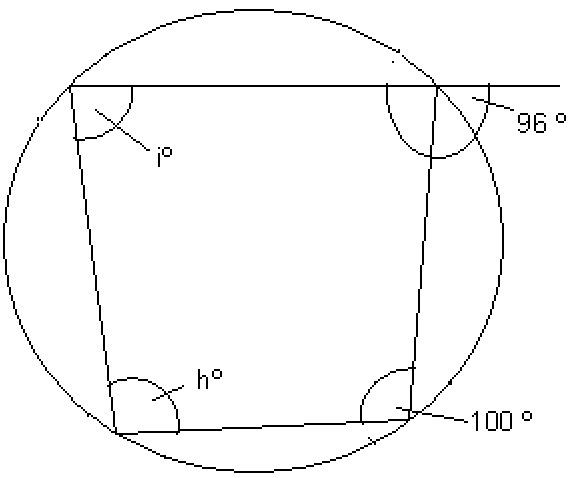
4.



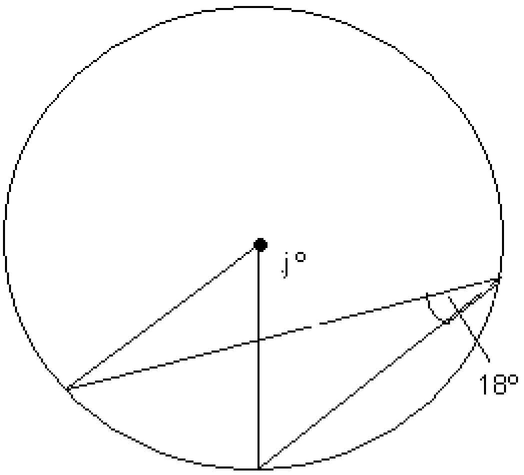
5.



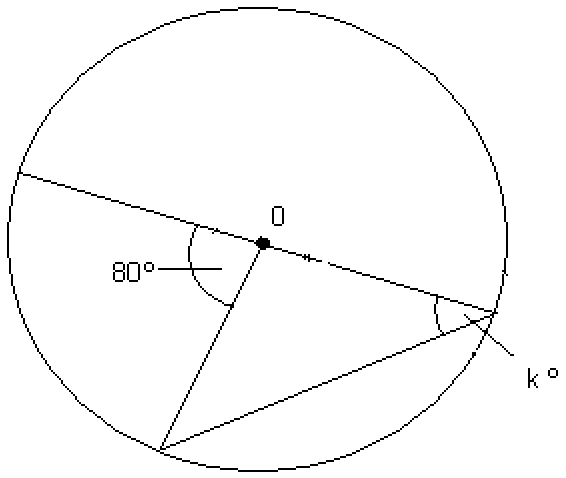
6.



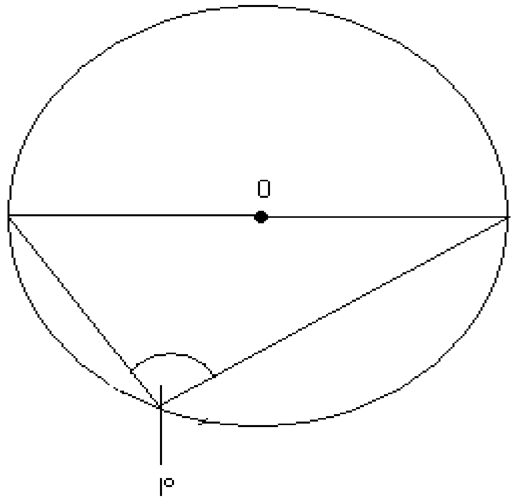
7.



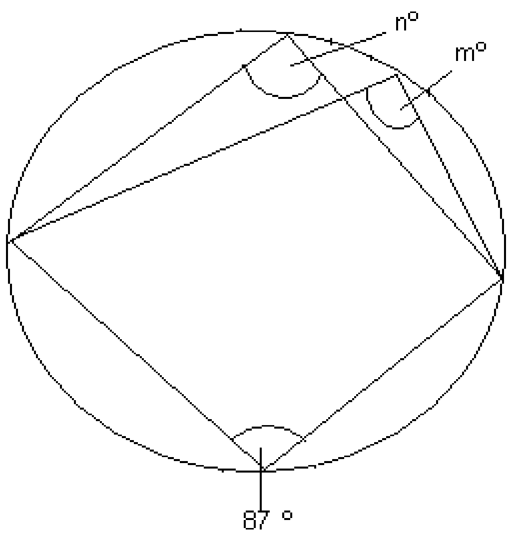
8.



9.

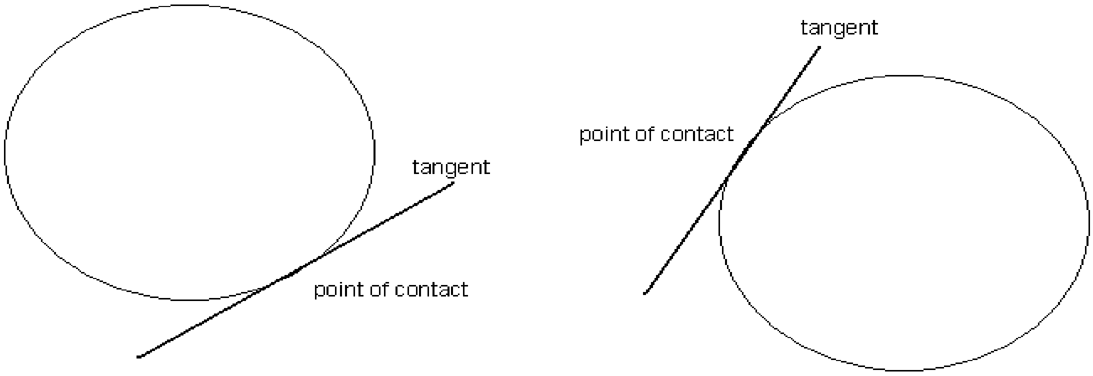


10.



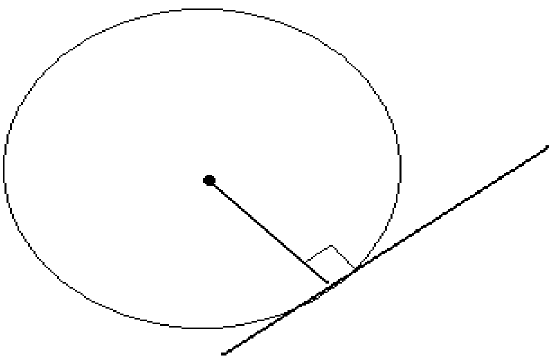
TANGENT PROPERTIES OF A CIRCLE

A tangent is a line which touches a circle at one point only.

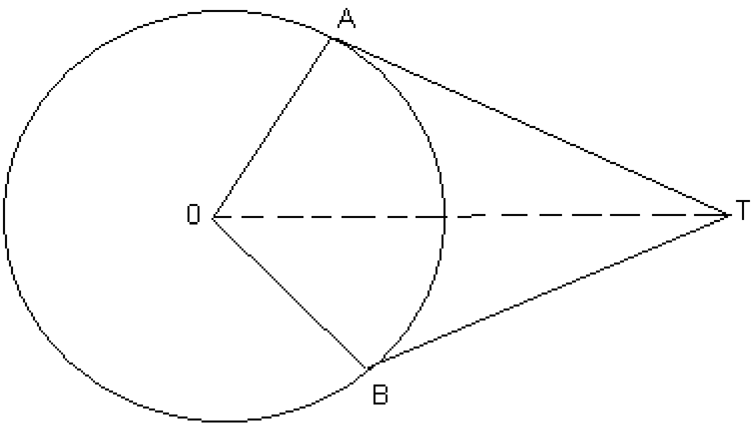


Rules applying to tangents

1. The angle between a tangent and a radius at the point of contact which is 90º.

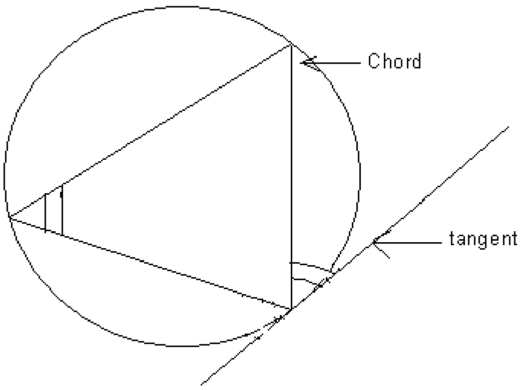


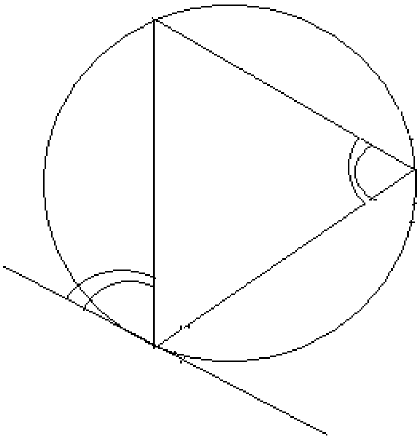
2. Are drawn to a circle from an external point, then two tangents are equal in length.



AT = TB
and <ATO =<BTO

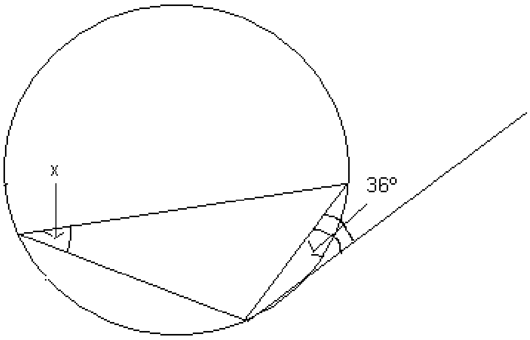
3. The angle between the tangent and the chord equals the angle in the alternate segment. This is called the **alternate segment therom**.





Example 1

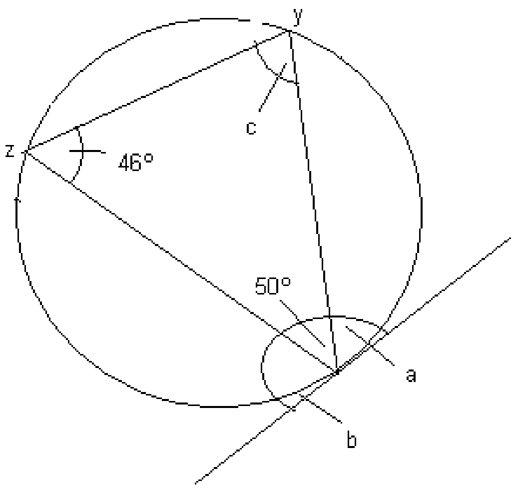
Find X



X = 36 (alternate segment theorem)

Example 2

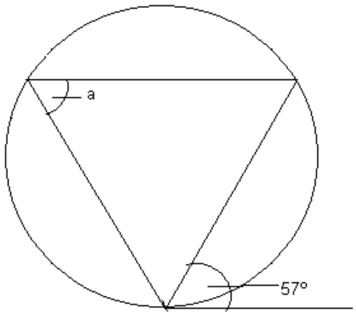
Find a, b and c.



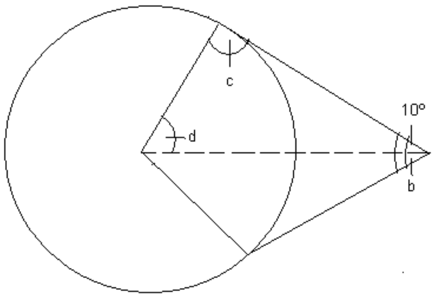
a = 46º (alternate segment theorem)
b = 84º (angles on a straight line add up to 180º) (180º – 50º – 46º = 84º)
c = 84º (alternate segment theorem)

Exercise 2

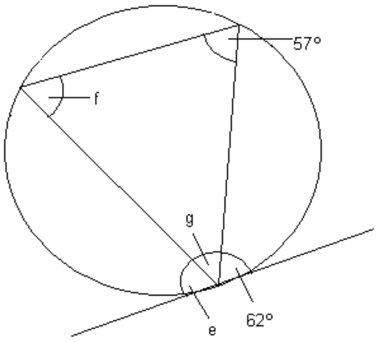
1.



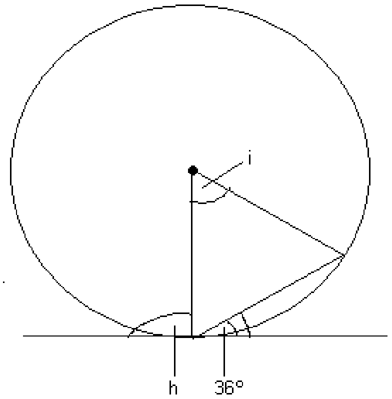
2.



3.



4.



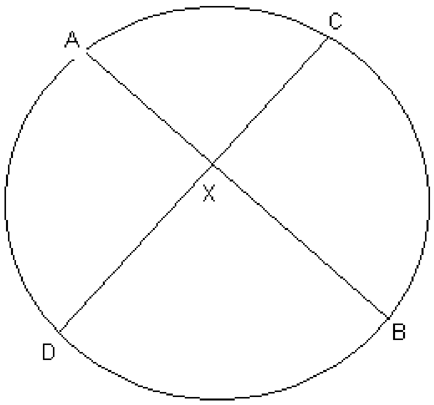
THINK!

Extra lines needed.

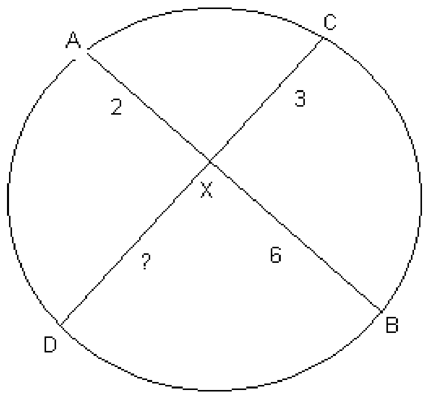
INTERSECTING CHORDS

The following rules must be learnt!

1. AX.XB = CX.XD

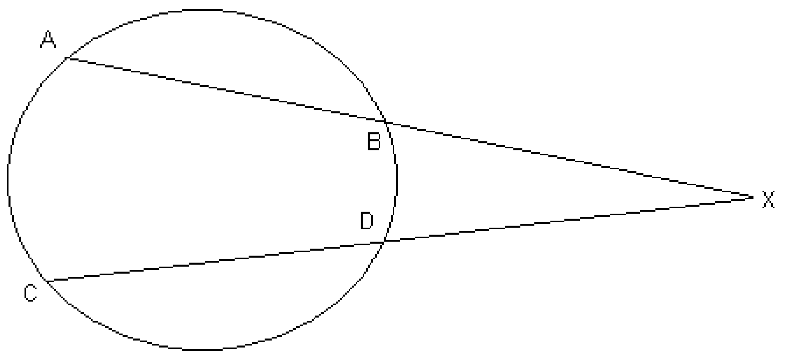


Example



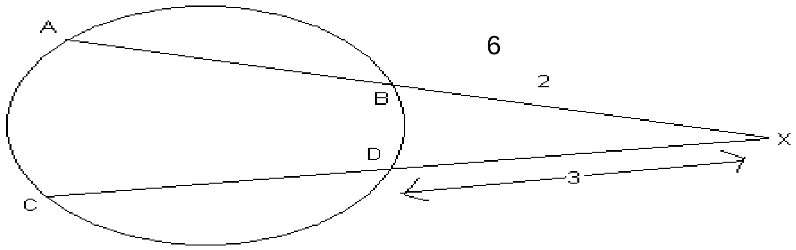
AX.XB = CX.XD
2 x 6 = 3 x XD
$\frac{12}{3}$ = XD
4=XD

2. AX.XB = CX.XD



Example

Find CD

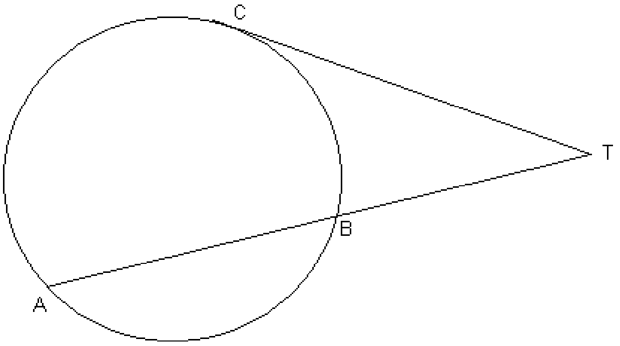


AX.XB = CX.XD
6 X 2 = CX.3

$\frac{12}{3}$ = CX
4=CX

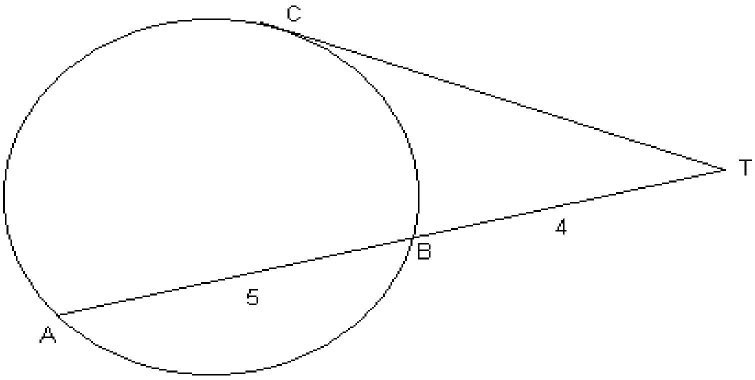
CD = CX – DX
CD = 4 – 3 CD = 1

3. $CT^2 = AT.BT$



Example 1

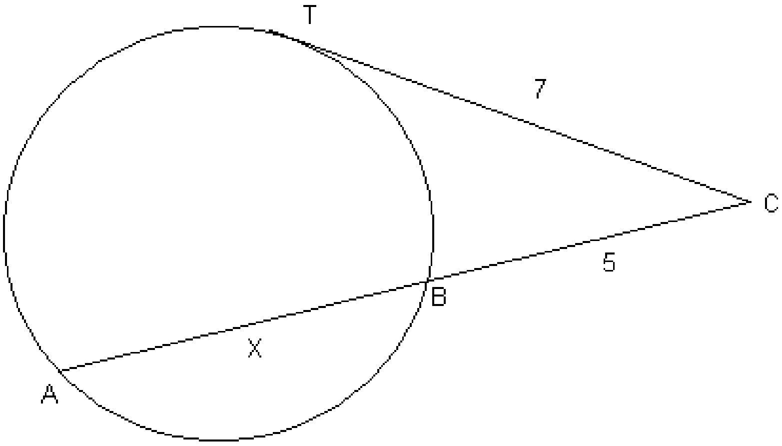
Find CT



$CT^2 = AT.BT$ ($AT = 5 + 4$) $CT^2 = 9 \times 4 = 36$
$CT = 6$

Example 2

Find CA



Let AB be X $AC = 5 + X$

$CT^2 = AC.BC$ $7^2 = (5 + X)5$

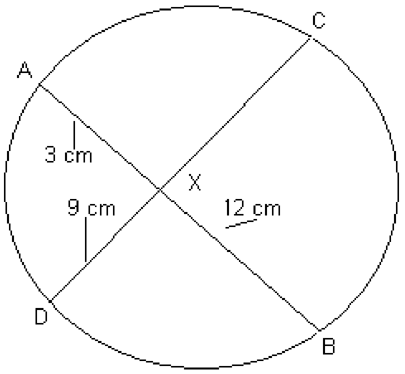
$49 = 25 + 5X \qquad 49 - 25 = 5X$

$24 = 5X \qquad \frac{24}{5} = X$

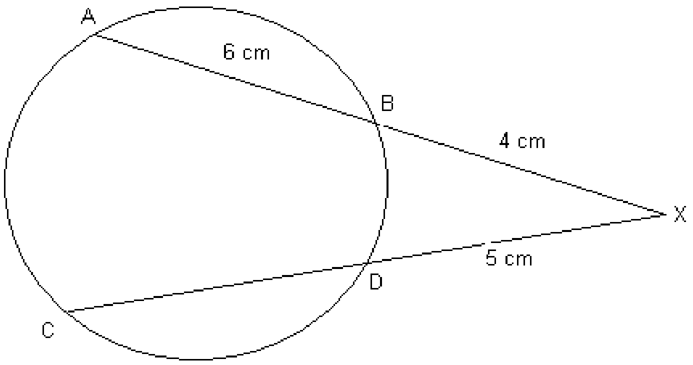
$4\frac{4}{5} = X \qquad CA = 5 + 4\frac{4}{5} = 9\frac{4}{5}$

Exercise 3

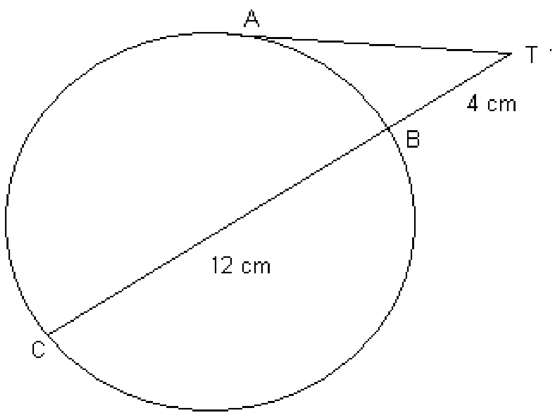
1. Find CX



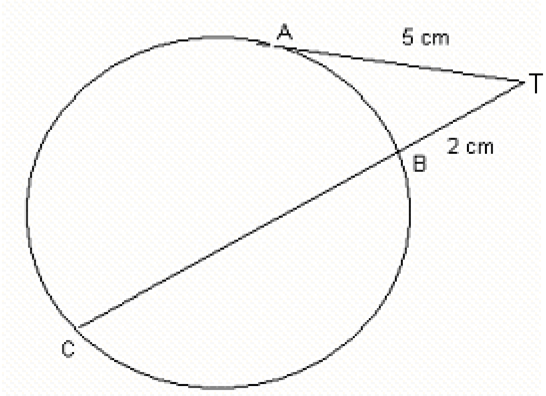
2. Find CD



3. Find AT



4. Find BC



ANSWERS

Exercise 1

1.	a = 56º	2.	b = 90º, c= 58º
3.	d = 36º	4.	e = 130º
5.	f = 112º, g = 76º	6.	h=96º, i=80º
7.	j = 36º	8.	k = 40º
9.	l = 90º	10.	m = 93º, n = 93º

Exercise 2

TANGENTS

1. a = 57º
2. b = 10º, c = 90º, d = 80º
3. e = 57º, f = 62º, g = 61º
4. h = 90º, i = 72º

Exercise 3

INTERSECTING CHORDS

1.	CX = 4cm	2.	CD = 3cm
3.	AT = 8cm	4.	CB = 10.5 cm