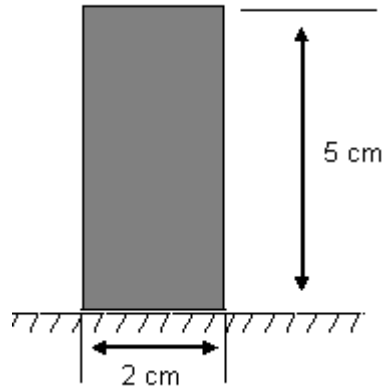


AREA OF A TRIANGLE

Let's start off with something nice and simple, like finding the area of the rectangle shown opposite.



Exercise 1

Find the area of a rectangle with the side lengths shown above.

$$\begin{aligned} \text{Area} &= \quad \times \\ &= \quad \text{cm}^2 \end{aligned}$$

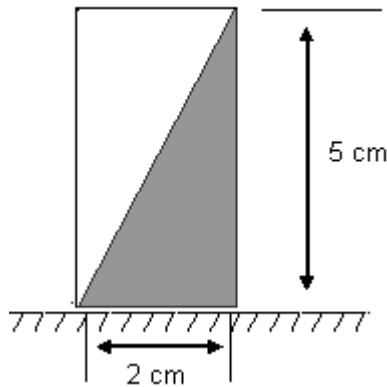
Always check your answers, even if you are confident with your own solution, as few “pearls of wisdom” may have been added to supplement the main text.

Now check your answer.

Since the rectangle, shown above, is standing on the 2cm long side, we could call this its base and the 5cm long side its height.

Therefore: Area of this rectangle = base x height.

Now let's cut our rectangle in half by drawing a diagonal as shown opposite.



Exercise 2

Calculate the area of shaded portion of the rectangle shown above.

Area of whole rectangle = X

= _____ cm^2

Area of shaded portion = $\frac{1}{2} X$

= _____ cm^2

Now check your answer.

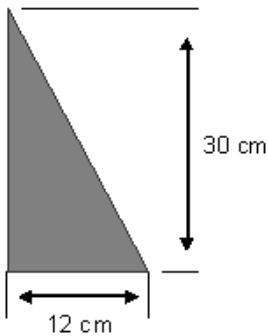
Earlier, we said that the area of this rectangle was its base multiplied by its height. This time the area of the shaded portion was half the base multiplied by the height. But the shaded area is the **TRIANGLE**, so:

Area of triangle = $\frac{1}{2}$ (base x height)

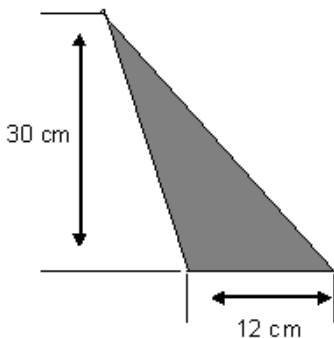
Exercise 3

Calculate the areas of the following triangles.
All dimensions are in millimetres.

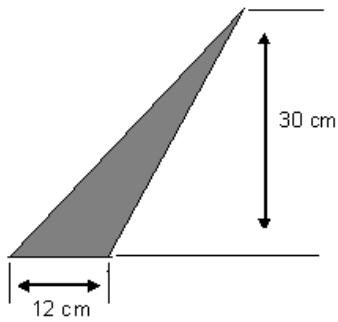
a)



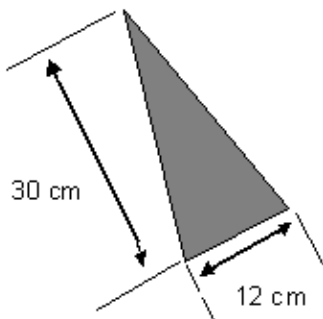
b)



c)



d)



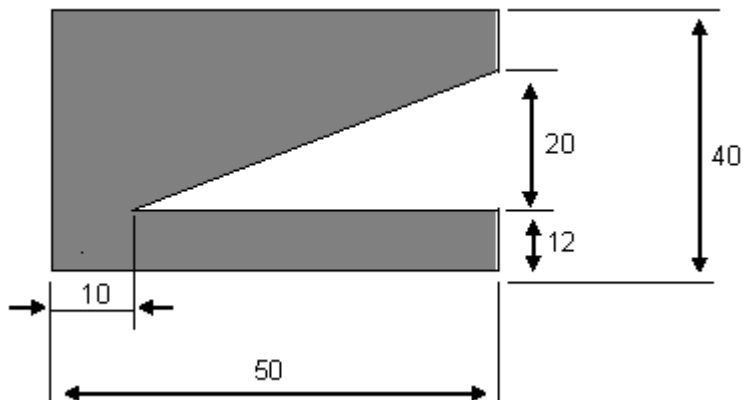
Now check your answers.

The next two activities will require rather more thought.

Exercise 4

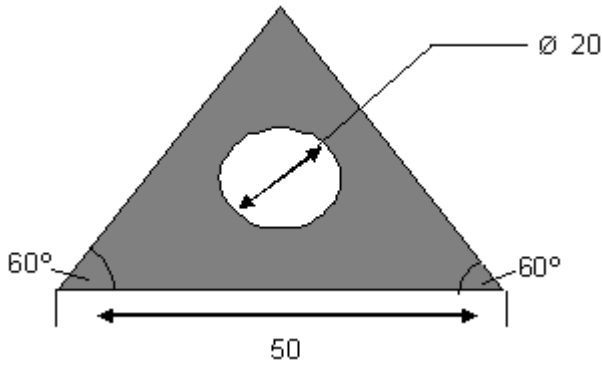
Calculate the area of the shaded portions of the following figures:

a)



Dimensions in millimetres.

b)



Remember: Ø20 means 20mm diameter and that the area of a

$$\text{Circle} = \frac{\pi D^2}{4}$$

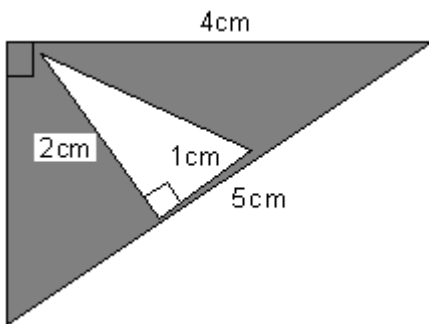
Dimensions millimetres.

Now check your answers.

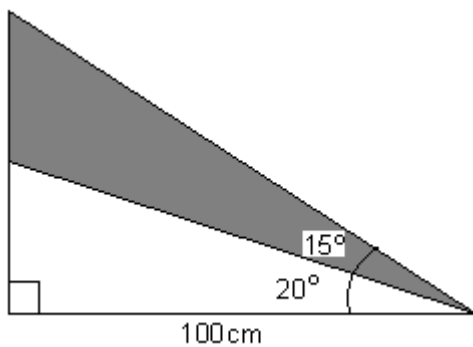
Exercise 5

Calculate the area shaded portions of the following figures:

a)



b)



Now check your answers.

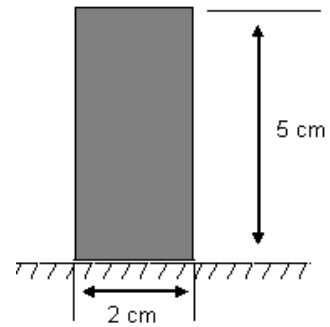
ANSWERS

Exercise 1

Area of a rectangle = length X breadth

$$\text{Area} = 5 \text{ cm} \times 2 \text{ cm}$$

$$= 10 \text{ cm}^2$$



You could, of course have the got same answer by saying:

“standing” on the 2 cm end, the Area = base x height.

Now return to the text.

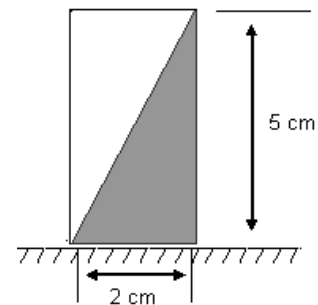
Exercise 2

Area of whole rectangle = 2cm x 5cm

$$= 10\text{cm}^2$$

$$\text{Area of shaded portion} = \frac{1}{2} \times 10 \text{ cm}$$

$$= 5 \text{ cm}^2$$



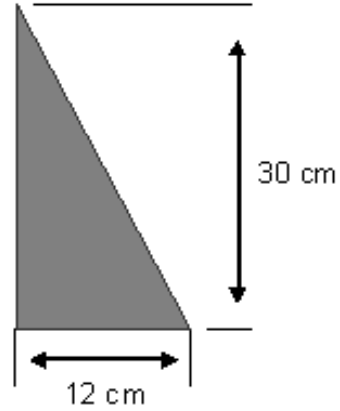
You have just found the area of the triangle by a rather circuitous route.

We will use this information to determine an expression for finding the area of a triangle more elegantly.

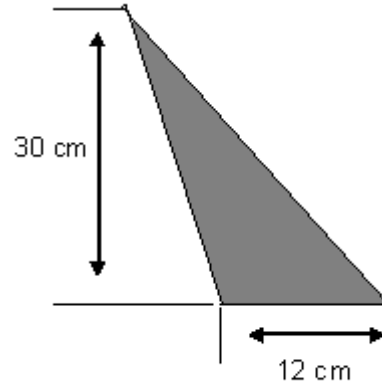
Now return to the text.

Exercise 3

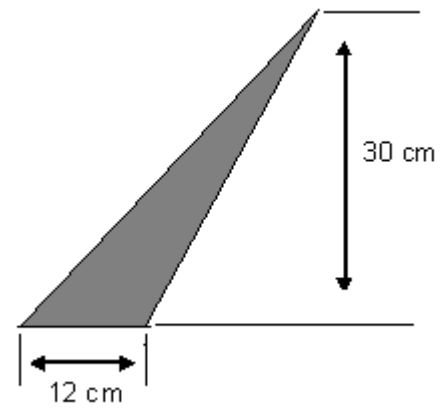
a) $\text{Area} = \frac{1}{2} (\text{base} \times \text{height})$
 $= \frac{1}{2} \times 12\text{mm} \times 30\text{mm}$
 $= 180 \text{ mm}^2$



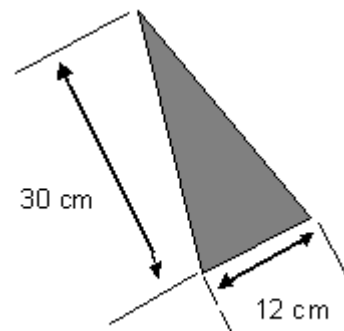
b) $\text{Area} = \frac{1}{2} (\text{base} \times \text{height})$
 $= \frac{1}{2} \times 12\text{mm} \times 30\text{mm}$
 $= 180 \text{ mm}^2$



c) $\text{Area} = \frac{1}{2} (\text{base} \times \text{height})$
 $= \frac{1}{2} \times 12\text{mm} \times 30\text{mm}$
 $= 180 \text{ mm}^2$



d) $\text{Area} = \frac{1}{2} (\text{base} \times \text{height})$
 $= \frac{1}{2} \times 12\text{mm} \times 30\text{mm}$
 $= 180 \text{ mm}^2$

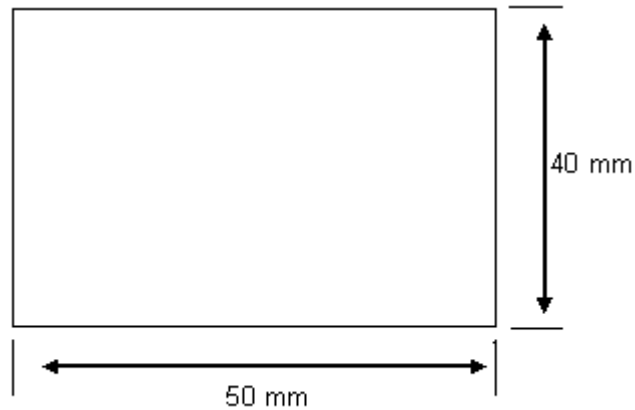


- The shape or position of the triangle has **NO** effect on the area.
- The height is always measured perpendicular to the base. (At right-angles to the base).

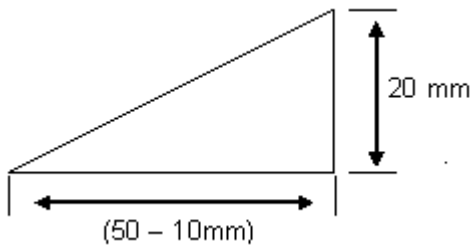
Now return to the text.

Exercise 4

a)



$$\text{Area of rectangle} = 50 \times 40 = 2000\text{mm}^2$$



$$\text{Area of triangle} = \frac{1}{2} \times 40 \times 20 = 400\text{mm}^2$$

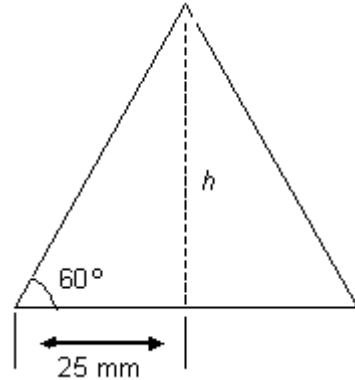
Area of shaded portion = area of rectangle minus the area of triangle.

$$\begin{aligned} \text{Area of shaded portion} &= 2000\text{mm}^2 - 400\text{mm}^2 \\ &= 1600\text{mm}^2 \end{aligned}$$

- b) First, we use trigonometry to find the height of the triangle.

Since the triangle is symmetrical, a perpendicular line (h) from the base to the apex bisects the base.

$$\begin{aligned} h &= 25 \times \tan 60^\circ \\ &= 25 \times 1.7321 \\ &= 43.303 \text{ mm} \end{aligned}$$



$$\text{Area of triangle} = \frac{1}{2} (\text{base} \times \text{height})$$

$$\begin{aligned} &= \frac{1}{2} \times 50 \times 43.303 \\ &= 1082.6 \text{ mm}^2 \end{aligned}$$

$$\text{Area of hole} = \frac{\pi D^2}{4}$$

$$= \frac{3.1416 \times 20^2}{4}$$

$$\text{Area of shaded portion of figure} = \text{area of triangle} - \text{area of hole}$$

$$= 1082.6 - 314.26$$

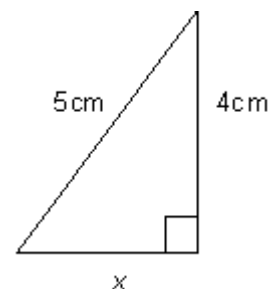
$$= 768.44 \text{ mm}^2$$

Now return to the text.

Exercise 5

- a) First we will stand the large right-angled triangle on its short side. The short side becomes the base. The perpendicular height is 4cm, and we can find the base (x) using Pythagoras' theorem.

$$\begin{aligned} 4^2 + x^2 &= 5^2 \\ x^2 &= 5^2 - 4^2 \\ &= 25 - 16 \\ &= 9 \\ \therefore x &= \sqrt{9} \\ &= 3 \end{aligned}$$



$$\begin{aligned} \text{Area of large triangle} &= \frac{1}{2} (\text{base} \times \text{height}) \\ &= \frac{1}{2} \times 3 \times 4 \\ &= 6 \text{ cm}^2 \end{aligned}$$

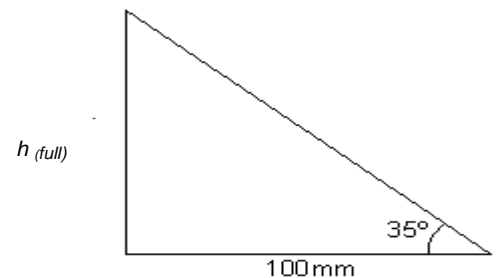
$$\begin{aligned} \text{Area of small triangle} &= \frac{1}{2} (\text{base} \times \text{height}) \\ &= \frac{1}{2} \times 1 \times 2 \\ &= 1 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of shaded portion} &= 6 \text{ cm}^2 - 1 \text{ cm}^2 \\ &= 5 \text{ cm}^2 \end{aligned}$$

b) There are two ways of finding the shaded area.

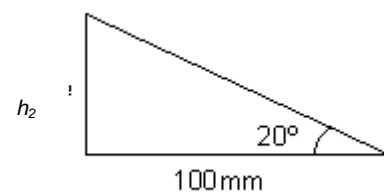
Method 1

$$\begin{aligned} h_{\text{full}} &= \frac{100}{1} \times \tan 35^\circ \\ &= 100 \times 0.7002 \\ &= 70.02 \text{ m} \end{aligned}$$



$$\begin{aligned} \text{Area of large triangle} &= \frac{1}{2} (\text{base} \times \text{height}) \\ &= \frac{1}{2} \times 100 \times 70.02 \\ &= 3501 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} h_2 &= 100 \times \tan 20^\circ \\ &= 100 \times 0.36397 \\ &= 36.397 \text{ m} \end{aligned}$$



$$\begin{aligned} \text{Area of small triangle} &= \frac{1}{2} (\text{base} \times \text{height}) \\ &= \frac{1}{2} \times 100 \times 36.397 \\ &= 1819.9 \text{ m}^2 \end{aligned}$$

Area of shaded portion of the figure.

$$\begin{aligned} h_{(full)} - h_2 &= 3501 - 1819.9 \\ &= 1681.1 \text{ m}^2 \end{aligned}$$

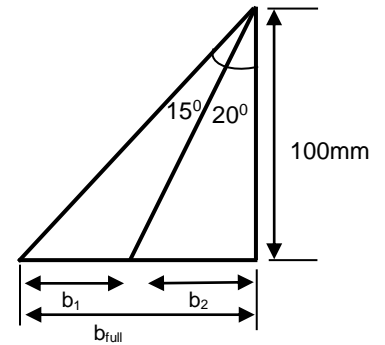
Method 2

Turn the triangles up on end as shown opposite.

$$\begin{aligned} b_1 &= 100 \tan 35^\circ \\ &= 70.02 \text{ m} \end{aligned}$$

$$\begin{aligned} b_2 &= 100 \tan 20^\circ \\ &= 36.4 \text{ m} \end{aligned}$$

$$\begin{aligned} b_1 &= b_{full} - b_2 \\ &= 33.62 \text{ m} \end{aligned}$$



Area of shaded portion of the figure.

$$= \frac{1}{2} (\text{base} \times \text{height})$$

$$= \frac{1}{2} \times 33.62 \times 100$$

$$= 1681 \text{ m}^2$$

Any slight difference in the answers is due to rounding off at various stages in the calculation.
If you are having serious difficulties have a chat with your tutor.