SECOND DERIVATIVE

Suppose $y = f(x) = 3x^2$. Then $\frac{dy}{dx} = f'(x) = 6x$

$\frac{dy}{dx}$ (or $f'(x)$) is sometimes called the 'first derivative'.

Below is the second derivative, known for short as "dee two wye dee exe squared".

$$\frac{d^2y}{dx^2} = f''(x) = 6$$

$f''(x)$ is spoken as "eff double dashed exe" or "eff double dash exe".

Example

In the following examples,

Write $\frac{dy}{dx}$ = … etc. If the question says " y = … ",

Write $f'(x)$ = … etc. If the question says " $f(x)$ =…

Write $\frac{dy}{dx} = f'(x)$ = … etc. If the question says $y = f(x)$ =…

Exercise

Find the formula for the second derivative in the following cases:

a) $y = 3x^2 + 5x - 9$ b) $f(x) = x^4 - 4x^3 + 2$

c) $f(x) = 2x^{-3} + x^2$ d) $y = f(x) = 4x^5 - 3x^3$

e) $g(x) = 7x^{-1}$ f) $y = ax^3$ (a is a constant.)

Now check your answers.

You can go on differentiating as much as you like. The third derivative is called "dee three wye dee exe cubed", and is written as $\frac{d^3y}{dx^3}$ or $f'''(x)$.

$\frac{d^6y}{dx^6}$ corresponds to $f''''''(x)$, and so on. But you are unlikely to need to go beyond the second derivative in this course.

SUMMARY

$\frac{dy}{dx}$ or $f'(x)$ is known as the first derivative. When you differentiate the first derivative you get the second derivative.

The second derivative is called “dee two wye dee exe squared” or “eff double dashed exe”: $\frac{d^2y}{dx^2}$ or $f''(x)$.

If you differentiate again, you get the third derivative, and so on.

ANSWERS

Exercise

a) $y = 3x^2 + 5x - 9$

$\frac{dy}{dx} = 6x + 5$

$\frac{d^2y}{dx^2} = 6$

The equation just said '$y =$ ' , not '$f(x) =$ ' . If the equation has said '$y = f(x) =$ … ' , then the final statement would have been

$\frac{d^2y}{dx^2} = f''(x) = 6$

b) $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

c) $f(x) = 2x^{-3} + x^2$

$f'(x) = -6x^{-4} + 2x$

$f''(x) = 24x^{-5} + 2$

d) $y = f(x) = 4x^5 - 3x^3$

$\frac{dy}{dx} = f'(x) = 20x^4 - 9x^2$

$\frac{d^2y}{dx^2} = f''(x) = 80x^3 - 18x$

e) $g(x) = 7x^{-1}$

$g'(x) = -7x^{-2}$

$g''(x) = 14x^{-3}$

f) $y = ax^3$

$\frac{dy}{dx} = 3ax^2$

$\frac{d^2y}{dx^2} = 6ax$

Now return to the text.