FRACTIONS
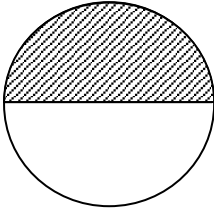
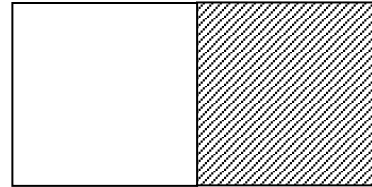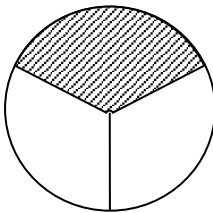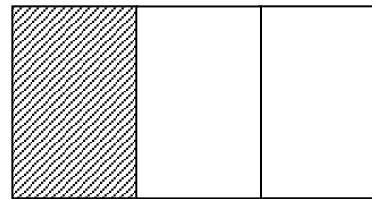**WHAT IS A FRACTION?**

A fraction is a part of a whole

1.

This shaded area is one part out of 2. Written ½, said 'one-half'.

2.

This shaded area is one part out of 3. Written 1/3, said 'one-third'.

3.
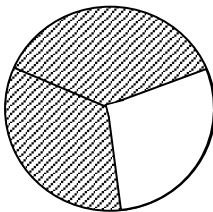
This shaded area is two parts out of 3. Written 2/3, said 'two-thirds'.
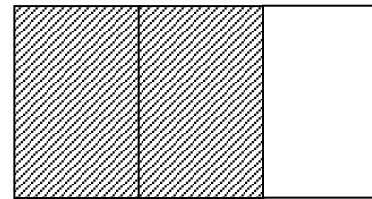
As you can see, these figures are divided into **equal** parts.

If all the parts are different, as in the example below (not equal parts), then the shaded area is not a third.

NOT THIRDS

However, the diagram below is divided into eight equal parts, so the shaded area is 'three-eights'.

$$\frac{3}{8}$$

A fraction has   a top number   **Numerator**
                                     AND
                                     A bottom number   **Denominator**

The line shows that the top number (the numerator) is **divided by** the bottom number (the denominator).
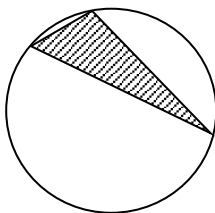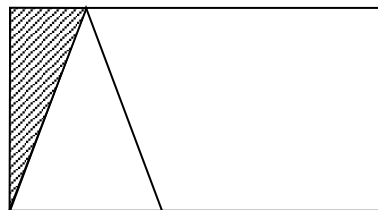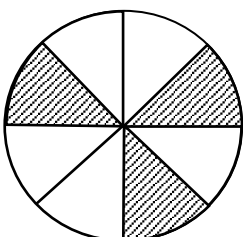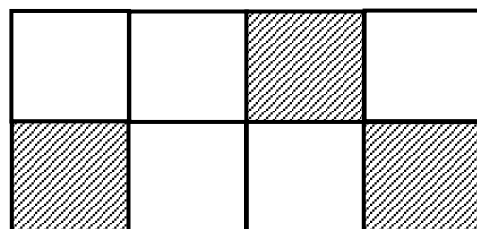
There are **three** types of fractions:

1.   **Common** (or **vulgar** or **proper**) fractions where the numerator is **less than** the denominator.

       e.g. $\dfrac{1}{2}$, $\dfrac{3}{4}$, $\dfrac{1}{8}$

2.   **Improper** (sometimes called **'top heavy'** fractions) where the numerator is **greater than** the denominator.

       e.g. $\dfrac{5}{2}$, $\dfrac{7}{3}$, $\dfrac{19}{7}$

3.   **Mixed numbers**, where the number has **whole** numbers and parts of whole numbers (common fractions).

       e.g. $12\dfrac{1}{2}$, $345\dfrac{4}{9}$, $27\dfrac{11}{12}$

Remember, the value of a fraction is unchanged if the numerator and the denominator are **BOTH** multiplied by or are **BOTH** divided by the same number; for example:-

       $\dfrac{30}{40}$ is the same as $\dfrac{3}{4}$ (both top and bottom divided by 10)

       $\dfrac{3}{27}$ is the same as $\dfrac{1}{9}$ (both top and bottom divided by 3)

       $\dfrac{2}{3}$ is the same as $\dfrac{4}{6}$ (both top and bottom multiplied by 2)

       $\dfrac{1}{7}$ is the same as $\dfrac{6}{42}$ (both top and bottom multiplied by 6)

What is the numerator in the next example?

       $\dfrac{3}{4} = \dfrac{?}{28}$

You must ask yourself by what have you multiplied the 4 (three-quarters) to get to 28. The answer is 7, so you must multiply the numerator (the three), by the same number (7).

So, 3 x 7 = 21:

The new numerator is 21:

$$\frac{3}{4} = \frac{21}{28}$$

What is the new denominator here?

$$\frac{6}{27} = \frac{2}{?}$$

By what have you divided 6 to get to 2?

Answer is by 3, so you must now divide 27 by 3 to give you the new denominator.
27 ÷ 3 = 9 (new denominator)

So, $\frac{6}{27} = \frac{2}{9}$

Now fill in these gaps.

Exercise 1

1. $\frac{2}{7} = \frac{?}{14}$ 	2. $\frac{4}{5} = \frac{?}{25}$ 	3. $\frac{3}{8} = \frac{6}{?}$

4. $\frac{3}{10} = \frac{?}{40}$ 	5. $\frac{1}{7} = \frac{5}{?}$ 	6. $\frac{4}{12} = \frac{1}{?}$

7. $\frac{10}{25} = \frac{?}{5}$ 	8. $\frac{9}{15} = \frac{3}{?}$ 	9. $\frac{8}{64} = \frac{?}{8}$

10 $\frac{18}{48} = \frac{3}{?}$

REDUCING A FRACTION TO ITS LOWEST TERMS
(You may know this as 'cancelling')

Example 1
reduce $\frac{10}{12}$ to its lowest terms

First ask which number goes into **both** 10 and 12 (without a remainder, of course!)?

Answer is 2, so divide both numerator and denominator by 2.
What does this give you?

$$\frac{10}{12} \div \frac{2}{2} = \frac{5}{6}$$

As there is no number which 'goes into' 5 and 6 this fraction cannot be reduced, so you have now finished cancelling.

Example 2

reduce $\frac{132}{198}$ to its lowest terms

1. Divide top and bottom by 2, to give

$$\frac{132}{198} \div \frac{2}{2} = \frac{66}{99}$$

2. Divide the new fraction by 11

$$\frac{66}{99} \div \frac{11}{11} = \frac{6}{9}$$

3. Now divide this by 3

$$\frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

This will reduce no more, and so it is the answer!

Exercise 2

1. $\frac{9}{18}$ 2. $\frac{15}{25}$ 3. $\frac{42}{48}$ 4. $\frac{180}{240}$ 5. $\frac{210}{315}$

CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS

Example 1

$2\frac{1}{4}$ is a mixed number

How do I change this into an improper fraction?

1. Multiply the whole number by the denominator 2 x 4 = 8

2. Add this to the numerator 8 + 1 = 9
3. The denominator stays the same
4. The new numerator, 9, is placed over the 4 giving $\dfrac{9}{4}$

Example 2

change $5\dfrac{3}{7}$ to an improper fraction.

1. Multiply the whole number 5, by the denominator 7, 5 x 7 = 35
2. Add to this the numerator 35 + 3 = 38
3. Put this over the original denominator, giving $\dfrac{38}{7}$ (answer)

Example 3

$4\dfrac{3}{10}$ to an improper fraction is $\dfrac{43}{10}$

Go through the steps to check how this answer was reached, then try the examples.

Exercise 3

1. $3\dfrac{1}{2}$ 2. $4\dfrac{2}{5}$ 3. $3\dfrac{7}{8}$ 4. $15\dfrac{1}{2}$ 5. $3\dfrac{5}{9}$

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS

Example 1

$\dfrac{22}{7}$ Divide the numerator by the denominator 22 ÷7 = 3 remainder 1. The 3 is the whole number and the remainder is the new numerator. The denominator stays the same.

Answer = $3\dfrac{1}{7}$

Example 2

$\dfrac{7}{2}$ as a mixed number?

Divide the 7 by the 2, this gives 3 remainder 1. The 3 is the whole number, the remainder is the numerator.

Answer = $3\dfrac{1}{2}$

Exercise 4

1. $\dfrac{13}{2}$ 2. $\dfrac{23}{7}$ 3. $\dfrac{18}{5}$ 4. $\dfrac{29}{11}$ 5. $\dfrac{53}{12}$

MULTIPLICATIONS OF FRACTIONS

Example 1

$$\frac{1}{2} \quad \text{x} \quad \frac{1}{3}$$

Steps to take to multiply

1. See if any numbers will cancel in this case, none will.
2. Multiply the numerators (top numbers) to give the new numerator 1 x 1 = 1.
3. Multiply the denominators (bottom numbers) to give the new denominator 2 x 3 = 6.

So the answer is $\dfrac{1 \times 1}{2 \times 3} = \dfrac{1}{6}$

Example 2

$$\frac{2}{3} \times \frac{3}{4}$$

1. Will any numbers cancel? Yes 2 will go into 4. Then 3 will go into 3.

CANCEL

2. Now multiply the numerators (1 x 1 = 1)
3. Multiply the denominators (1 x 2 = 2)

Answer $= \dfrac{1}{2}$

The main point to remember is that, when you are cancelling, you can cancel **any numerator** with **any denominator:**

E.g.

to give $\dfrac{1}{1} \times \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{8}$

Do not worry if you have not cancelled fully at this stage – you will be able to reduce the fraction to its lowest terms at the end of your calculation.

MULTIPLICATION WITH MIXED NUMBERS

Example 1

$$3\frac{1}{2} \times \frac{4}{7}$$

FIRST STEP – change mixed numbers to **IMPROPER FRACTIONS.**

So, $3\frac{1}{2}$ becomes $\frac{7}{2}$

The sum now looks like this:

$$\overset{1}{\underset{1}{\cancel{\frac{7}{2}}}} \times \overset{2}{\underset{1}{\cancel{\frac{4}{7}}}} = \frac{1}{1} \times \frac{2}{1} = \frac{2}{1}$$

Answer = 2

Notice that cancelling takes place as usual **after** you have changed the mixed numbers to improper fractions.

Example 2

$$1\frac{4}{9} \times 2\frac{1}{2} \quad \text{becomes} \quad \frac{13}{9} \times \frac{5}{2} = \frac{65}{18} = 3\frac{11}{18}$$

Or set the sum out as:

$$1\frac{4}{9} \times 2\frac{1}{2} = \frac{13}{9} \times \frac{5}{2} = \frac{65}{18}$$

$$= 3\frac{11}{18}$$

Example 3

$$2 \times \frac{3}{7} \quad \text{2 is written as '2 over 1' } \textbf{or } \frac{2}{1} \text{ so the sum can be written as:}$$

$$\frac{2}{1} \times \frac{3}{7} = \frac{6}{7} \text{ or set the sum out as:}$$

$$2 \times \frac{3}{7} = \frac{2}{1} \times \frac{3}{7} = \frac{6}{7}$$

Example 4

$$\frac{7}{8} \times 4 \quad \text{becomes} \quad \overset{}{\underset{2}{\cancel{8}}}\frac{7}{} \times \overset{1}{\cancel{4}}\frac{}{1} \quad \text{Cancel 4 and 8}$$

$$= \frac{7}{2} \times \frac{1}{1} = \frac{7}{2} = 3\frac{1}{2} \text{ or set the sum out as: } \frac{7}{8} \times 4 = \frac{7}{2} \times \frac{1}{1} = \frac{7}{2} = 3\frac{1}{2}$$

Example 5

$$5 \times 2\frac{3}{7} \text{ becomes } \frac{5}{1} \times \frac{17}{7} = \frac{85}{7} = 12\frac{1}{7}$$

Example 6

What is two thirds of four and a half? You know that 'of' means multiply, so you can now write this sum as follows:

$$\overset{1}{\frac{\cancel{2}}{\cancel{3}}} \times \overset{3}{\frac{\cancel{9}}{\cancel{2}}} = \frac{1}{1} \times \frac{3}{1} = \frac{3}{1}$$

Now try Exercise 5, use the examples given above to help you.

Exercise 5

1. $\frac{3}{5} \times \frac{4}{7}$ 2. $\frac{3}{5} \times \frac{5}{9}$ 3. $\frac{3}{5} \times \frac{1}{3}$ 4. $2\frac{1}{2} \times \frac{4}{5}$

5. $3\frac{1}{2} \times 4\frac{1}{7}$ 6. $3 \times \frac{2}{7}$ 7. $\frac{3}{5} \times 5$ 8. $2 \times 1\frac{1}{2} \times \frac{1}{3}$

9. $3\frac{3}{4} \times 1\frac{3}{5} \times 1\frac{1}{8}$ 10. $\frac{4}{5}$ of $1\frac{1}{2}$

DIVISION OF FRACTIONS

Remember that you must change **mixed numbers** to **improper fractions**

1. The first fraction remains the same.
2. Change the division sign to a multiplication sign.
3. Invert (or turn upside down) the second fraction.
4. Carry on as you would for multiplication of fractions.

So using the steps above:

Example 1

$$\frac{2}{3} \div \frac{5}{7}$$

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

Example 2

$$1\frac{4}{5} \div \frac{2}{3} \quad \text{becomes} \quad \frac{9}{5} \times \frac{3}{2} = \frac{27}{10} = 2\frac{7}{10}$$

Example 3

$$1\frac{4}{5} \div 2\frac{1}{3} \quad \text{becomes} \quad \frac{9}{5} \div \frac{7}{3} = \frac{9}{5} \times \frac{3}{7} = \frac{27}{35}$$

Example 4

$$4 \text{ divided by } \frac{3}{4} \quad \text{becomes} \quad \frac{4}{1} \times \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

Example 5

$$2\frac{1}{2} \text{ divided by } 10 = \frac{5}{2} \div \frac{10}{1}$$

$$= \frac{\overset{1}{\cancel{5}}}{2} \times \frac{1}{\underset{2}{\cancel{10}}} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

Exercise 6

1. $\dfrac{3}{4} \div \dfrac{1}{2}$ 2. $\dfrac{5}{6} \div \dfrac{2}{3}$ 3. $2\dfrac{1}{2} \div 1\dfrac{1}{4}$ 4. $5\dfrac{1}{3} \div \dfrac{4}{9}$ 5. $\dfrac{5}{12} \div 2\dfrac{1}{2}$

6. $1\dfrac{1}{2} \div \dfrac{3}{7}$ 7. $1\dfrac{3}{4} \div \dfrac{1}{8}$ 8. $1\dfrac{1}{6} \div 4\dfrac{2}{3}$ 9. $\dfrac{3}{5} \div \dfrac{9}{20}$ 10. $3\dfrac{1}{3} \div \dfrac{5}{9}$

ADDITION OF FRACTION

Just as we can add 4 bananas to 3 bananas (7 bananas) we can add 4 twelfths to 3 twelfths (7 twelfths). Similarly, if we try to add 4 bananas to 3 oranges we still have 4 bananas and 3 oranges, then we cannot add 4 fifths to 3 sevenths.

You can only add (or subtract) fractions **when the denominators have the same number.**
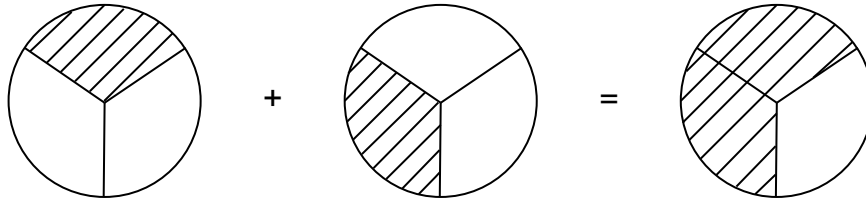
Just as $\dfrac{1}{3} + \dfrac{1}{3} = \dfrac{1+1}{3} = \dfrac{2}{3}$

Notice that only the numerators are added!

$$\frac{1}{5} \; + \; \frac{2}{5} \; = \; \frac{3}{5}$$

Two or more fractions which have **different denominators** cannot be added until you alter them so that the denominators are the same.

You will now use your knowledge of **equivalent fractions** and **lowest common multiple!**

Example 1

$\dfrac{1}{2} + \dfrac{1}{3}$ cannot be added yet until the denominators have the same number. Ask which is the lowest number into which both 2 and 3 will divide exactly? 6 is the lowest number.

You know that $\dfrac{1}{2}$ is also $\dfrac{3}{6}$ and that $\dfrac{1}{3}$ is also $\dfrac{2}{6}$

Now that you have the equivalent fractions with the same denominator, the adding can take place,
$\dfrac{3}{6} + \dfrac{2}{6} = \dfrac{5}{6}$

Only Numerators Are Added!

Example 2

$\dfrac{2}{7} + \dfrac{3}{4}$ 28 is the lowest number into which 4 and 7 will divide.

$\dfrac{2}{7}$ can also be written $\dfrac{8}{28}$ and $\dfrac{3}{4}$ can also be written $\dfrac{21}{28}$

Now that the denominators are the same the two fractions can be added to give:

$$\frac{8}{28} + \frac{21}{28} = \frac{8+21}{28} = \frac{29}{28} = 1\frac{1}{28}$$

If you have **mixed numbers,** there are various methods which you can use. We suggest that the following way may be the simplest:

Example 3

$$1\frac{1}{2} + 2\frac{1}{3}$$

1. **Add** the whole numbers

$$3 + \frac{1}{2} + \frac{1}{3}$$

2. Now carry on as before for the fraction parts

$$3\frac{3}{6} + \frac{2}{6} = 3\frac{3+2}{6} = 3\frac{5}{6}$$

Make Sure that you write the whole number each time (otherwise, you may forget it) **and** that you write it **bigger** than the fraction

Example 4

$$\frac{2}{3} + \frac{4}{9}$$

$$= \frac{6}{9} + \frac{4}{9}$$

$$= \frac{10}{9} = 1\frac{1}{9}$$

Example 5

$$2\frac{3}{4} + 1\frac{1}{2}$$

$$12\frac{3}{4} + 1\frac{2}{4}$$

$$= 13\frac{5}{4}$$

$$= 14\frac{1}{4}$$

Example 6

$$4\frac{1}{2} + 3\frac{5}{6} + 2\frac{1}{3}$$

$$= 9\frac{3}{6} + \frac{5}{6} + \frac{2}{6}$$

$$= 9\frac{3+5+2}{6}$$

$$= 9\frac{10}{6}$$

$$= 9 + 1\frac{4}{6}$$

$$= 9 + 1\frac{2}{3}$$

$$= 10\frac{2}{3}$$

Exercise 7

1. $\dfrac{3}{4} + \dfrac{1}{2}$ 2. $\dfrac{1}{7} + \dfrac{2}{3}$ 3. $\dfrac{3}{4} + \dfrac{1}{8}$ 4. $\dfrac{4}{5} + \dfrac{2}{3}$

5. $\dfrac{3}{4} + \dfrac{4}{5}$ 6. $5\dfrac{1}{2} + 1\dfrac{1}{4}$ 7. $4\dfrac{3}{5} + 1\dfrac{1}{4}$ 8. $7\dfrac{1}{3} + 1\dfrac{1}{12}$

9. $4\dfrac{1}{3} + \dfrac{2}{3}$ 10. $5\dfrac{3}{8} + 1\dfrac{3}{4} + 4\dfrac{7}{8} + 3\dfrac{5}{16}$

SUBTRACTION OF FRACTIONS

As in addition of fractions, the denominators of the fractions **must** be the same;

Example 1

$$\frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

only the numerators are subtracted!

Example 2

$\dfrac{1}{5}$ - $\dfrac{1}{10}$ $\dfrac{1}{5}$ can be written as $\dfrac{2}{10}$ so we now have $\dfrac{2}{10}$ - $\dfrac{1}{10}$ = $\dfrac{1}{10}$

Example 3

$\dfrac{3}{4}$ - $\dfrac{2}{3}$

LCM is 12 $\dfrac{3}{4}$ = $\dfrac{9}{12}$ and $\dfrac{2}{3}$ = $\dfrac{8}{12}$ so we can now write $\dfrac{9}{12}$ - $\dfrac{8}{12}$ = $\dfrac{9-8}{12}$ = $\dfrac{1}{12}$

When you have **mixed numbers** we suggest the following method:

1. Change mixed numbers to improper fractions:

   $1\dfrac{5}{6}$ - $\dfrac{1}{3}$ becomes $\dfrac{11}{6}$ - $\dfrac{1}{3}$

2. Carry on as before – finding **LCM** and so the equivalent fractions, giving

   $\dfrac{11}{6}$ - $\dfrac{2}{6}$ = $\dfrac{11-2}{6}$ = $\dfrac{9}{6}$ = $1\dfrac{3}{6}$ = $1\dfrac{1}{2}$

Example 4

$3\dfrac{3}{4}$ - $2\dfrac{7}{8}$

= $\dfrac{15}{4}$ - $\dfrac{23}{8}$

= $\dfrac{30}{8}$ - $\dfrac{23}{8}$

= $\dfrac{30-23}{8}$

= $\dfrac{7}{8}$

Example 5

6 - $\dfrac{2}{3}$

= $\dfrac{6}{1}$ - $\dfrac{2}{3}$

= $\dfrac{18}{3}$ - $\dfrac{2}{3}$

$$= \frac{18 - 2}{3}$$

$$= \frac{16}{3}$$

$$= 5\frac{1}{3}$$

Exercise 8

1. $\dfrac{7}{8} - \dfrac{3}{8}$ 　　2. $\dfrac{2}{7} - \dfrac{1}{14}$ 　　3. $\dfrac{1}{2} - \dfrac{3}{8}$ 　　4. $\dfrac{4}{5} - \dfrac{3}{10}$

5. $\dfrac{3}{11} - \dfrac{1}{22}$ 　　6. $1\dfrac{1}{2} - \dfrac{2}{3}$ 　　7. $3\dfrac{1}{3} - \dfrac{5}{12}$ 　　8. $2\dfrac{5}{12} - 1\dfrac{7}{12}$

9. $3\dfrac{3}{4} - 1\dfrac{7}{8}$ 　　10. $5 - \dfrac{3}{8}$

QUESTIONS WITH BOTH ADDING AND SUBTRACTING

Example 1

$$2\frac{1}{2} - 1\frac{1}{8} + 3\frac{1}{4}$$

In this case convert **all** the mixed numbers to improper fractions, and carry on as before, taking care **not to confuse** the signs

$$= \frac{5}{2} - \frac{9}{8} + \frac{13}{4}$$

$$= \frac{20}{8} - \frac{9}{8} + \frac{26}{8}$$

$$= \frac{20 - 9 + 26}{8}$$

$$= \frac{11 + 26}{8}$$

$$= \frac{37}{8} = 4\frac{5}{8}$$

Exercise 9

1. $2\dfrac{1}{4} - \dfrac{1}{8} + 2\dfrac{1}{2}$ 　　2. $\dfrac{1}{5} - \dfrac{1}{10} + \dfrac{1}{20}$ 　　3. $2\dfrac{1}{6} - 1\dfrac{5}{6} + \dfrac{7}{12}$

4. $1\dfrac{1}{4} + \dfrac{1}{2} - \dfrac{5}{8}$ 5. $5\dfrac{1}{10} - 3\dfrac{1}{2} + 1\dfrac{1}{4}$

ANSWERS

Exercise 1

1. $\dfrac{4}{14}$
2. $\dfrac{20}{25}$
3. $\dfrac{6}{16}$
4. $\dfrac{12}{40}$
5. $\dfrac{5}{35}$

6. $\dfrac{1}{3}$
7. $\dfrac{2}{5}$
8. $\dfrac{3}{5}$
9. $\dfrac{1}{8}$
10. $\dfrac{3}{8}$

Exercise 2

1. $\dfrac{1}{2}$
2. $\dfrac{3}{5}$
3. $\dfrac{7}{8}$
4. $\dfrac{3}{4}$
5. $\dfrac{2}{3}$

Exercise 3

1. $\dfrac{7}{2}$
2. $\dfrac{22}{5}$
3. $\dfrac{31}{8}$
4. $\dfrac{31}{2}$
5. $\dfrac{32}{9}$

Exercise 4

1. $6\dfrac{1}{2}$
2. $3\dfrac{2}{7}$
3. $3\dfrac{3}{5}$
4. $2\dfrac{7}{11}$
5. $4\dfrac{5}{12}$

Exercise 5

1. $\dfrac{12}{35}$
2. $\dfrac{1}{3}$
3. $\dfrac{1}{5}$
4. 2
5. $14\dfrac{1}{2}$

6. $\dfrac{6}{7}$
7. 3
8. 1
9. $6\dfrac{3}{4}$
10. $1\dfrac{1}{5}$

Exercise 6

1. $1\dfrac{1}{2}$
2. $1\dfrac{1}{4}$
3. 2
4. 12
5. $\dfrac{1}{6}$

6. $3\dfrac{1}{2}$
7. 14
8. $\dfrac{1}{4}$
9. $1\dfrac{1}{3}$
10. 6

Exercise 7

1. $1\frac{1}{4}$ 2. $\frac{17}{21}$ 3. $\frac{7}{8}$ 4. $1\frac{7}{15}$ 5. $1\frac{11}{20}$

6. $6\frac{3}{4}$ 7. $5\frac{17}{20}$ 8. $8\frac{5}{12}$ 9. 5 10. $15\frac{5}{16}$

Exercise 8

1. $\frac{1}{2}$ 2. $\frac{3}{14}$ 3. $\frac{1}{8}$ 4. $\frac{1}{2}$ 5. $\frac{5}{22}$

6. $\frac{5}{6}$ 7. $2\frac{11}{12}$ 8. $\frac{5}{6}$ 9. $1\frac{7}{8}$ 10. $4\frac{5}{8}$

Exercise 9

1. $4\frac{5}{8}$ 2. $\frac{3}{20}$ 3. $\frac{11}{12}$ 4. $1\frac{1}{8}$ 5. $2\frac{17}{20}$