QUADRATIC EQUATIONS

A quadratic equation is an equation in which the highest power is a “squared” terms, e.g.

$c^2 – 2c + 1 = 0$
$c^2 – 3c = 4$
$c^2 - 6c = 0$
$4c^2-16 = 0$

are all quadratic equations.

They can be divided into two categories, which we are going to call “easy” quadratic equations, and “harder” ones.

The harder ones, which can be solved by using a **FORMULA**, will be dealt with later.
The easy ones will be looked at now.
To solve “easy” quadratic equations, certain rules should be followed.

1. Make the equation equal to zero, e.g.

   $c^2 – 3c = 4$
   becomes
   $c^2 - 3c – 4 = 0$

2. Make the term containing the square **POSITIVE,** i.e.

   $6 = 7c – 4c^2$
   becomes
   $4c^2 – 7c + 6 = 0$

3. Factorise the expression on the left hand side of the equation, e.g.

   $c^2 – 3c – 4 = 0$
   becomes
   $(c – 4)(c + 1) = 0$

4. If the product of these two factors is zero, then one factor or the other must be zero, i.e.

   $(c – 4) = 0$ or
   $(c + 1) = 0$
   Therefore, $c = +4$ or
   $c = - 1$
   Therefore, $c^2 – 3c = 4$
   $c^2 – 3c – 4 = 0$
   $(c – 4)(c +1) = 0$
   $c – 4 = 0$ or $c + 1 = 0$
   Therefore, $c = +4$ or $c = 1$

Example 1

$c^2 - 3c + 2 = 0$
$(c – 2)(c – 1) = 0$
So $c – 2 = 0$
or $c – 1 = 0$
$c = +2$ or $c = +1$

Example 2

$c^2 - 4 = 0$

Remember the difference between two squares?

$(c - 2)(c + 2) = 0$

$c - 2 = 0$ or $c + 2 = 0$

Therefore, $c = +2$ or $c = -2$

Example 3

$3c^2 - 12 = 0$

Take out **COMMON FACTORS** in this case, 3. The 3 has no effect on the solution.

$3(c^2 - 4) = 0$

$3(c - 2)(c + 2) = 0$

Therefore,

$c - 2 = 0$

or $c + 2 = 0$

So $c = +2$ or $c = -2$

Example 4

$c^2 - 3c = 0$

$c(c - 3) = 0$

$c = 0$

or $c - 3 = 0$

Therefore, $c = 0$ or $c = +3$

Example 5

$6 + 7c = -2c^2$

Re-arrange first, to make the squared term positive.

$2c^2 + 7c + 6 = 0$

$(2c + 3)(c + 2) = 0$

Therefore, $2c + 3 = 0$

Or $c + 2 = 0$

So $c = \frac{3}{2} = -1\frac{1}{2}$ or $c = -2$

As you saw above a quadratic equation always has two solutions - even though it is possible for both solutions to have the same numerical value, as the following example shows:

Example 6

$c^2 + 4c + 4 = 0$

$(c + 2)(c + 2) = 0$

Therefore, $c + 2 = 0$

Or $c + 2 = 0$

So $c = -2$ or $c = -2$

Example 7

Sometimes the question is already half done for you, as this example shows:

$(c + 2)(c – 6) = 0$
Therefore, $c + 2 = 0$
or $c – 6 = 0$
So $c = -2$ or $c = +6$

Exercise 1

1. $c^2 – 9 = 0$
2. $c^2 – c – 12 = 0$
3. $c^2 + c – 12 = 0$
4. $c^2 + 7c + 12 = 0$
5. $c^2 – 8c + 7 = 0$
6. $c^2 + 2c = 0$
7. $(c + 2)(c – 3) = 0$
8. $3c^2 + 9c = 0$
9. $3c^2 – 108 = 0$
10. $c^2 – 3c + 2 = 0$
11. $c^2 + c – 30 = 0$
12. $c^2 – 3c – 10 = 0$
13. $2c^2 – 7c + 5 = 0$
14. $2c^2 – 5c – 3 = 0$
15. $3c^2 – 7c + 2 = 0$

SOLUTION BY FORMULA

It is possible to be asked to solve a quadratic equation which will not factorise. In this case, you will have to use the formula (which is given to you in the examination). As soon as you see the words “solve to 2 decimal places” or “to 3 significant figures”, it means that you **HAVE TO USE** the formula.

(If you have any difficulty in factorising the examples in the previous part of this pack **YOU MAY ALSO USE THE FORMULA**. No marks will be deducted for using it but the formula method takes longer!)

The general form of a Quadratic Equation is given by:

$ac^2 + bc + c = 0$

where a represents the number and sign in front of the c^2 term
and b represents the number and sign in front of the c term
and c represents the number and sign at the end.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You do not need to know why – simply how to use it and make it work!

Note – All the numerators are divided by 2a.

As you can see, the sign before the square root sign is **“PLUS OR MINUS”,** which means that you will obtain two answers.

Example 1

$2c^2 - 3c - 7 = 0$

So, a = +2, b = -3, c = -7 **NB.** do not put c's after these numbers.

Now substitute these values for a, b and c in the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$$

It helps to put the numbers and signs in brackets, you are less likely to get signs wrong!

$$x = \frac{+3 \pm \sqrt{9 + 56}}{4}$$

$$x = \frac{+3 \pm \sqrt{65}}{4}$$

Now spilt up the answer, to give you two solutions

x is either $= \frac{+3 + 8.062}{4}$ or $\frac{+3 - 8.062}{4}$

$= \frac{11.062}{4}$ or $\frac{-5.062}{4}$ Careful signs!

= 2.7655 or -1.2655

Therefore x = 2.77 or -1.27 (correct to 2 dp)

Example 2

$c^2 + c - 1 = 0$

So, a = +1, b = +1, c = -1 (Remember that c can be written as 1c and so the coefficient of c = 1.)

SUBSTITUTE these values of a, b and c in the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(+1) \pm \sqrt{(+1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 4}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1 + 2.236}{2} \quad \text{or} \quad \frac{-1 - 2.236}{2}$$

$$x \text{ is either } \frac{1.236}{2} \quad \text{or} \quad \frac{-3.236}{2}$$

= 0.618 or - 1.618

Therefore x = 0.62 or - 1.62 (correct to 2dp)

Example 3

$2c(c + 1) – 3c(c – 2) = 6$ First multiply out the brackets. Careful signs!

$2c^2 + 2c – 3c^2 + 6c = 6$ (**NOTE** -3c x -2 = +6c)

Collect "like" terms together

$-c^2 + 8c = 6$

Re-arrange the equation so that the squared term is positive

$0 = c^2 – 8c + 6$

Now find a, b and c

a = +1, b = -8, c = +6 **SUBSTITUTE** these in the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{+8 \pm \sqrt{64 - 24}}{2}$$

$$x = \frac{+8 \pm \sqrt{40}}{2}$$

$$x = \frac{+8 + 6.325}{2} \quad \text{or} \quad \frac{+8 - 6.325}{2}$$

$$x = \frac{+14.325}{2} \quad \text{or} \quad \frac{+1.675}{2}$$

x = +7.1625 or +0.8375

Therefore

x = +7.16 or +0.84 (correct to 2 dp)

Exercise 2

Give your answers correct to 2 decimal places

1. $c^2 - 2c - 5 = 0$
2. $3c^2 + c - 1 = 0$
3. $2c^2 + 6c = 3$
4. $c(c + 2) = 5$
5. $c(c + 3) + 2c(c - 5) = 2$

ANSWERS

Exercise 1

1.	(c – 3)(c + 3) = 0	c = +3 or -3
2.	(c – 4)(c + 3) = 0	c = +4 or -3
3.	(c + 4)(c – 3) = 0	c = -4 or + 3
4.	(c + 4)(c + 3) = 0	c = -4 or -3
5.	(c – 7)(c – 1) = 0	c = +7 or +1
6.	c(c + 2) = 0	c = 0 or -2
7.	(c + 2)(c – 3) = 0	c = -2 or +3
8.	3c(c + 3) = 0	c = 0 or -3
9.	3(c – 6)(c+6) = 0	c = +6 or -6
10.	(c – 2)(c – 1) = 0	c = +2 or +1
11.	(c + 6)(c – 5) = 0	c = -6 or +5
12.	(c – 5)(c + 2) = 0	c = +5 or -2
13.	(2c – 5)(c – 1) = 0	c = +1 or 2.5
14.	(2c + 1)(c – 3) = 0	c = -0.5 or +3
15.	(3c – 1)(c – 2) = 0	c = $+\frac{1}{3}$ or +2

Exercise 2

USING THE FORMULA

1. $c^2 – 2c – 5 = 0$ a = 1, b = -2, c = -5

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4 + 20}}{2}$$

$$x = \frac{+2 \pm \sqrt{24}}{2}$$

$$x = \frac{+2 + 4.899}{2} \text{ or } \frac{+2 - 4.899}{2}$$

$$x = \frac{+6.899}{2} \text{ or } \frac{-2.899}{2}$$

x = +3..4495 or -1.4495

x = 3.45 or -1.45 (correct to 2dp)

2. $3c^2 + c - 1 = 0$ a = +3, b = +1, c = -1

$$x = \frac{-(+1) \pm \sqrt{(+1)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{1+12}}{6}$$

$$x = \frac{-1 \pm \sqrt{13}}{6}$$

$$x = \frac{-1+3.606}{6} \quad \text{or} \quad \frac{-1-3.606}{6}$$

$$x = \frac{2.606}{6} \quad \text{or} \quad \frac{-4.606}{6}$$

x = +0.4343 or -0.7677

x = +0.43 or -0.77 (correct to 2 dp)

3. $2c^2 + 6c = 3$

$2c^2 + 6c - 3 = 0$ a = +2, b = +6, c = -3

$$x = \frac{-(+6) \pm \sqrt{(+6)^2 - 4(+2)(-3)}}{2(+2)}$$

$$x = \frac{-6 \pm \sqrt{+36+24}}{4}$$

$$x = \frac{-6 \pm \sqrt{+60}}{4}$$

$$x = \frac{-6+7.746}{4} \text{ or } \frac{-6-7.746}{4}$$

$$x = \frac{+1.746}{4} \quad \text{or} \quad \frac{-13.746}{4}$$

x = 0.4365 or -3.4365

x = 0.44 or -3.44 (correct to 2dp)

4. $c(c + 2) = 5$

$c^2 + 2c = 5$

$c^2 + 2c - 5 = 0$ a = 1, b = 2, c = -5

$$x = \frac{-(+2) \pm \sqrt{(+2)^2 - 4(+1)(-5)}}{2(+1)}$$

$$x = \frac{-2 \pm \sqrt{+4 + 20}}{2}$$

$$x = \frac{-2 \pm \sqrt{+24}}{2}$$

$$x = \frac{-2 + 4.899}{2} \quad \text{or} \quad \frac{-2 - 4.899}{2}$$

$$x = \frac{+2.899}{2} \quad \text{or} \quad \frac{-6.899}{2}$$

$x = 1.4495$ or- 3.4495

$x = 1.45$ or- 3.45(correct to 2 dp)

5. $c(c + 3) + 2c(c - 5) = 2$

$c^2 + 3c + 2c^2 - 10c = 2$

$3c^2 - 7c = 2$

$3c^2 - 7c - 2 = 0$ a = +3, b = -7, c = -2

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(+3)(-2)}}{2(+3)}$$

$$x = \frac{+7 \pm \sqrt{+49 + 24}}{6}$$

$$x = \frac{+7 \pm \sqrt{+73}}{6}$$

$$x = \frac{+7 + 8.544}{6} \quad \text{or} \quad \frac{+7 - 8.544}{6}$$

$$x = \frac{+15.544}{6} \quad \text{or} \quad \frac{-1.544}{6}$$

$x = +2.5907$ or -02573

$x = +2.59$ or -0.26 (correct to 2 dp)