Investigation of Short Term Load Forecasts of Low Voltage Level Substation Feeders and the Effects of Weather

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LV workshop

4th LV workshop: Demand analytics and control for networks support, Reading, UK

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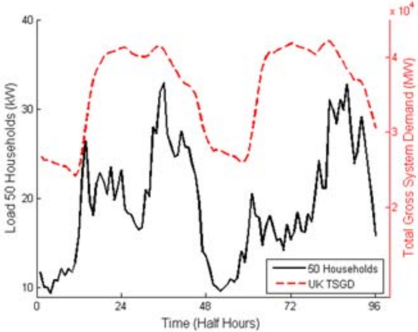
LV Load Data

- 100 Low Voltage Feeders in Bracknell, UK
- 83 of 100 purely residential, 8-109 customers (average 45)
- Hourly data: 31^{st} March 2014 to 22^{nd} November 2015
- Temperature and wind chill (hourly) forecasts/actuals. Begin 7am up to 96 hours ahead.

Challenges

- Data volatile (and less predictable) compared to higher voltages
- Varied number/types customers
- Impacts of temperature not fully understood
- Behaviour varies clearly over the day



Weather Forecasts: Properties

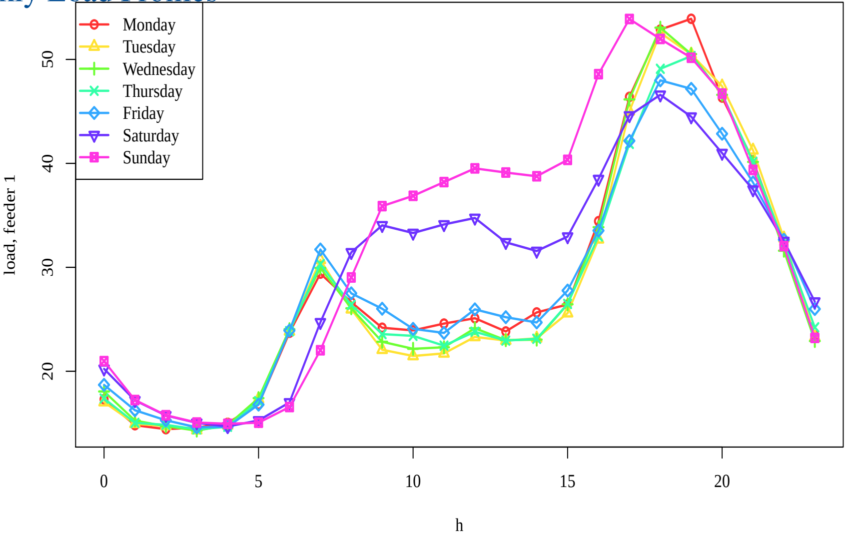
	Weather Variables	
Property	Temperature	Wind Chill
1 Day MAPE (%)	12	38
4 Day MAPE (%)	24	49
Correlation Load	-0.44	-0.44
Adj R^2 w/ load	0.23	0.24

- Forecast accuracy over testing period (9 to 16% Temp).

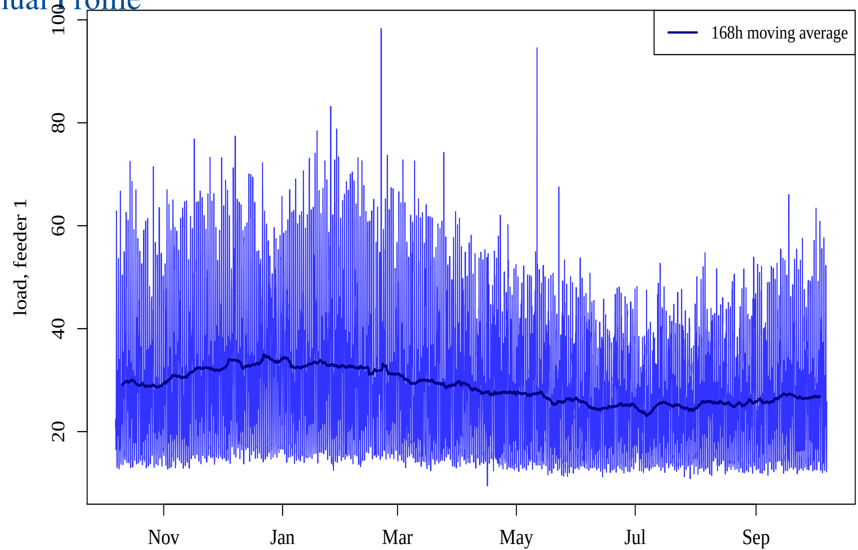
Required and open characteristic of LV load

- Seasonalities: Daily, weekly and annual
- Autoregressive effects
- ? *Model for separate hours vs. full model*
- ? *Trend*
- ? *Temperature effect*

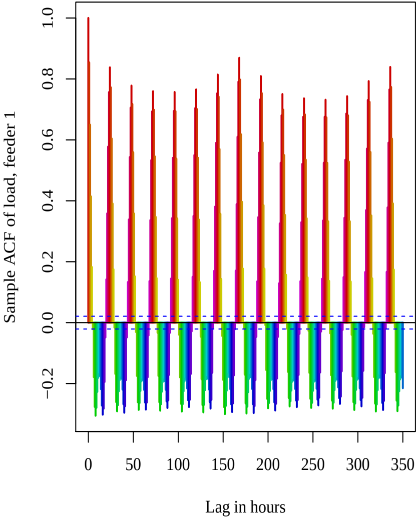
Weekly Load Profiles

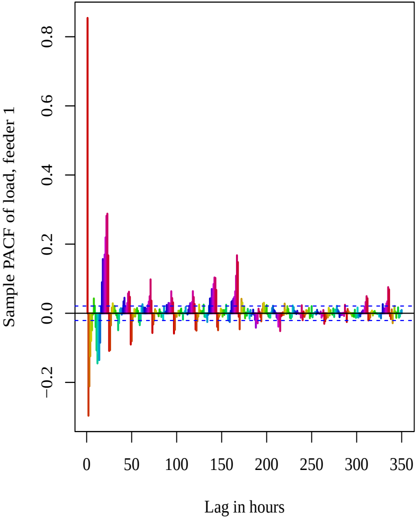


Annual Profile

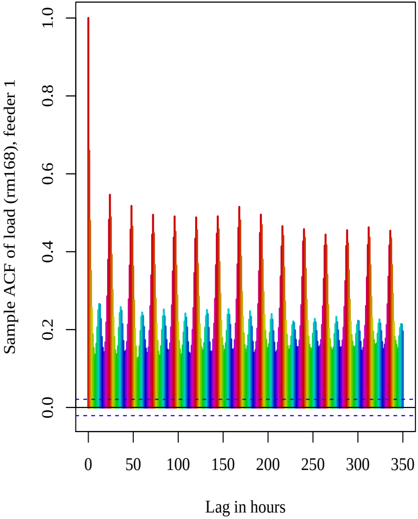


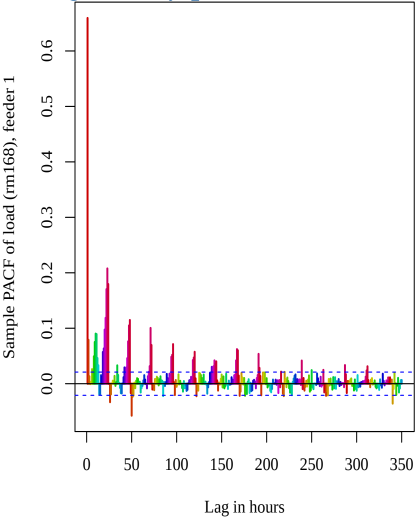
Autocorrelation structure of Load





Autocorrelation structure of Load (removing weekly profile)





Model 1: Seasonal Model

- Each hour is modelled separately $\Rightarrow$ 24 smaller models ($h = 1 + (t-1) \mod 24$)

$$L_t = a_0^h + a_1^h d(t) + \sum_{k=1}^{7} a_{2+k}^h \mathcal{K}_k(t) + \sum_{p=1}^{3} \mathcal{S}_p^h(t), \qquad (1)$$

where

- $d(t) = \left\lfloor \frac{t}{24} \right\rfloor + 1$ is the trend component (day of trial set)
- $\mathcal{K}_k(t)$ are dummy variables for the days of the week.
- $\mathcal{S}_p^h(t)$ is a seasonal term

$$\mathcal{S}_p^h(t) = b_p^h \sin\left(\frac{(2\pi p d(t))}{365}\right) + c_p^h \cos\left(\frac{(2\pi p d(t))}{365}\right), \qquad (2)$$

Model 1: Seasonal Model with temperature effects

Seasonal model with Weather terms:

$$L_t = a_0^h + a_1^h d(t) + \sum_{k=1}^{7} a_{2+k}^h \mathcal{K}_k(t) + \sum_{p=1}^{3} \mathcal{S}_p^h(t) + \sum_{q=1}^{3} f_q^h T(t)^q, \quad (3)$$

for some forecasted (or actual) weather variable $T(t)$ at time t.

- Point estimate generated from median quantile regression.
- We call this method **ST** when trend used and **SnT** without trend.
- Other methods tried
  - More seasonal terms
  - Weekend effect only
  - Mean (*faster*) vs. Median (*better*)

Model 2: Autoregressive Methods

- One time series is modelled $\Rightarrow$ one big model

Solve residual time series $r_t = L_t - \mu_t$, for *mean* profile μ_t

$$r_k = \sum_{k=1}^{p} \phi_k(r_{t-k}) + \epsilon_t. \tag{4}$$

- Autoregressive parameters $\phi_1, \ldots, \phi_p$ estimated using Burg method (Yule-Walker equation based $\Rightarrow$ stationary solution)
- Optimal order p by minimising AIC (Akaike Information Criterion) for $p \in \{0, \ldots, p_{\max} = 15S\}$, $S = 24$
- Mean μ_k estimated by OLS (ordinary least squares)

Model 2: Autoregressive Methods

Mean Model 1: Simple weekly average

$$\mu_t = \sum_{j=1}^{7S} \beta_j \mathcal{W}_j(t), \tag{5}$$

where

$$\mathcal{W}_j(t) = \begin{cases} 1, & t \mod 168 = j \\ 0, & \text{otherwise} \end{cases},$$

representing the hour j of the week.

- Extra term added to incorporate weather forecast/actuals
- We denote the model by **ARWD**($p_{\max}|n$)
  (n in-sample data size)

Model 2: Autoregressive Methods

Mean Model 2:

$$\mu_t = \sum_{j=1}^{7S} \beta_j \mathcal{W}_j(t) + \sum_{k=1}^{K} \alpha_{1,k} \sin(2\pi tk/A) + \alpha_{2,k} \cos(2\pi tk/A), \qquad (6)$$

with $A = 365.24 \times 24$, $K = 2$.

- We denote the model by **ARWDY**$(p_{\max}|n)$.
- Extra term added to incorporate weather forecast/actuals.

Model 3: Moving averages

- p-Week moving average:

$$L_t = \frac{1}{p} \sum_{k=1}^{p} L_{k-168k} + \epsilon_t \tag{7}$$

The model is denoted by **7SAV$p$weeks**.

- Special case: Weekly moving average last week as this (**7SAV1weeks**):

$$L_t = L_{k-168}, \tag{8}$$

We will denote this special case as **LW**.

Forecasting study

- Forecast Period: 1^{st} Oct 2015 to 22^{nd} Nov 2015.
- 1 to 96 (4 days) hours ahead forecast, rolling window.
- Forecast starts at 8am
- Errors measure

$$MAPE = \frac{100}{N} \sum_{t=1}^{N} \left| \frac{A_t - F_t}{A_t} \right| \tag{9}$$

- Test with real weather forecasts or actuals (Ex-ante vs Ex-post)

Errors Average

	MAPE % Weather Variables		
Method	None	Forecast	Actual
7SAV4Weeks	15.72	-	-
LW	19.11	-	-
ARWD(15S | 365S) Temp	14.65	20.17	20.03
ARWDY(15S | 365S) Temp	**14.64**	15.36	15.16
ST (Temp)	15.44	15.47	15.47
SnT (Temp)	15.77	15.79	15.80
ST (WChill)	15.44	15.47	15.47
SnT(WChill)	15.77	15.80	15.80

Errors Day to Day

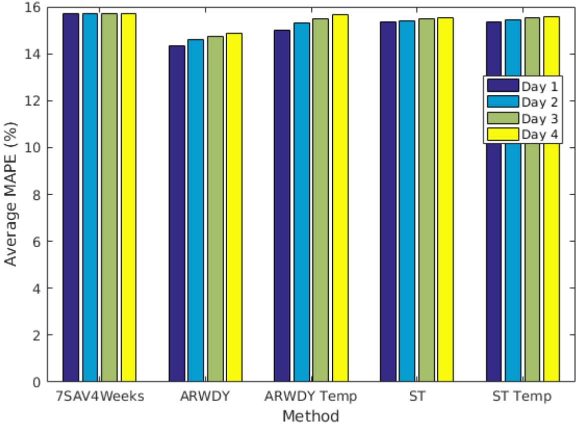


Figure : Comparison of Errors (MAPE) for different horizons including those using temperature forecasts.

Results

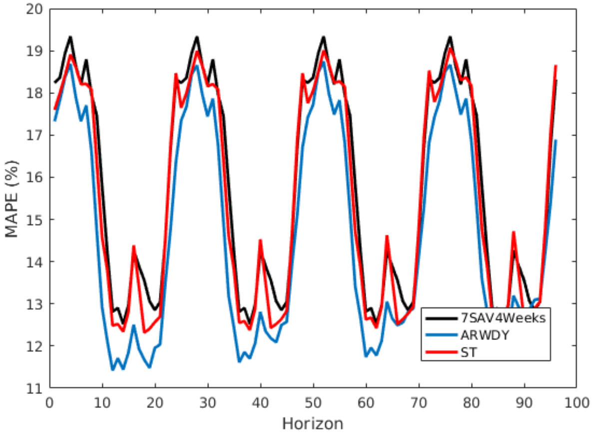
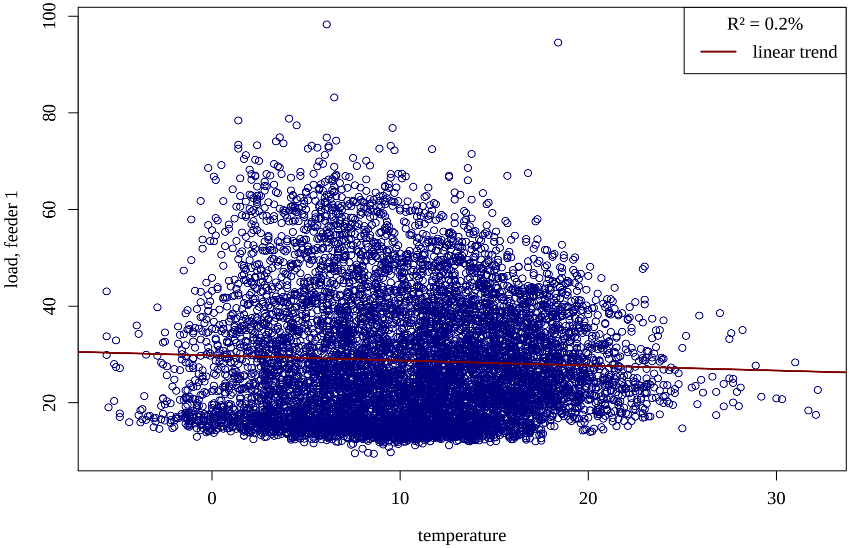
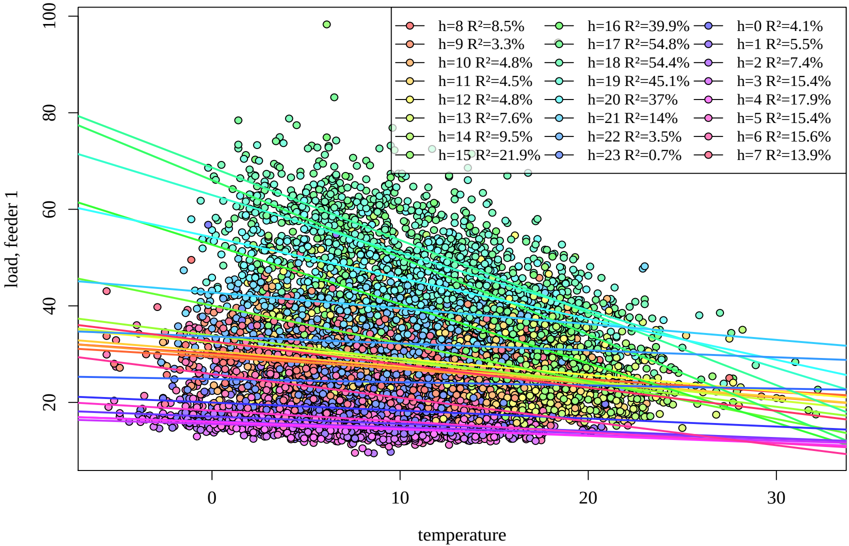
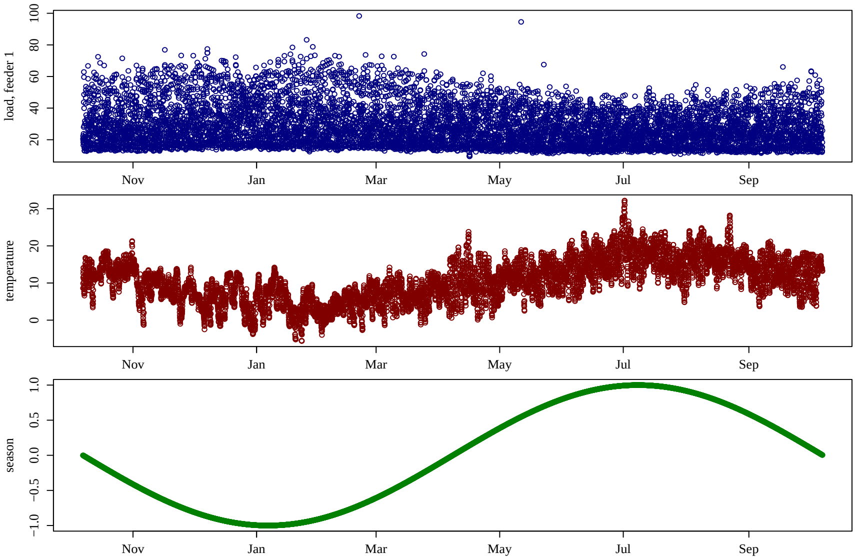


Figure : Comparison of Errors (MAPE) for different horizons.


R² = 0.2%
linear trend
load, feeder 1
temperature
100
80
60
40
20
0
10
20
30


h=8 R²=8.5%
h=9 R²=3.3%
h=10 R²=4.8%
h=11 R²=4.5%
h=12 R²=4.8%
h=13 R²=7.6%
h=14 R²=9.5%
h=15 R²=21.9%
h=16 R²=39.9%
h=17 R²=54.8%
h=18 R²=54.4%
h=19 R²=45.1%
h=20 R²=37%
h=21 R²=14%
h=22 R²=3.5%
h=23 R²=0.7%
h=0 R²=4.1%
h=1 R²=5.5%
h=2 R²=7.4%
h=3 R²=15.4%
h=4 R²=17.9%
h=5 R²=15.4%
h=6 R²=15.6%
h=7 R²=13.9%
load, feeder 1
100
80
60
40
20
0
10
20
30
temperature


load, feeder 1
20
40
60
80
100
Nov
Jan
Mar
May
Jul
Sep
temperature
0
10
20
30
season
−1.0
−0.5
0.0
0.5
1.0

The temperature is correlated with the 'season'

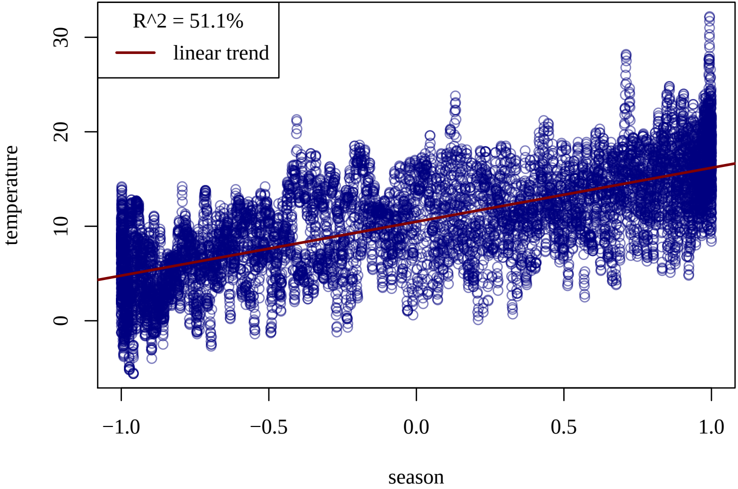
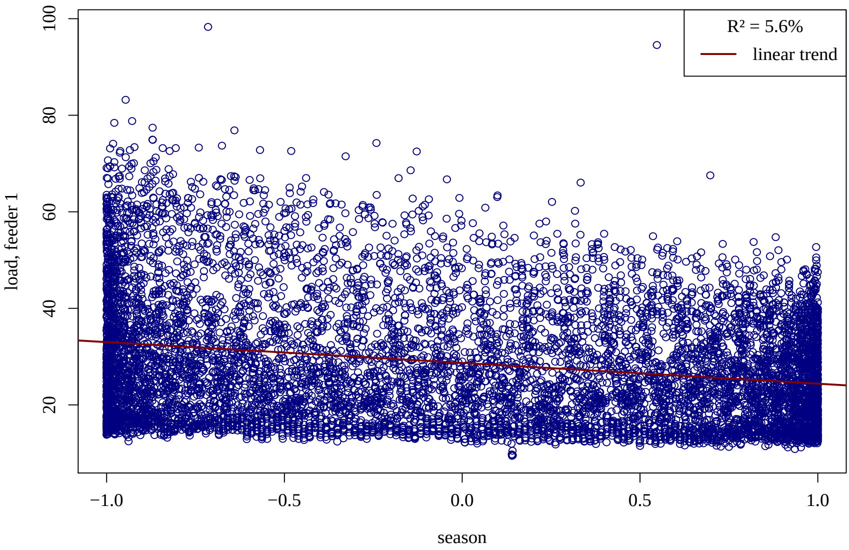
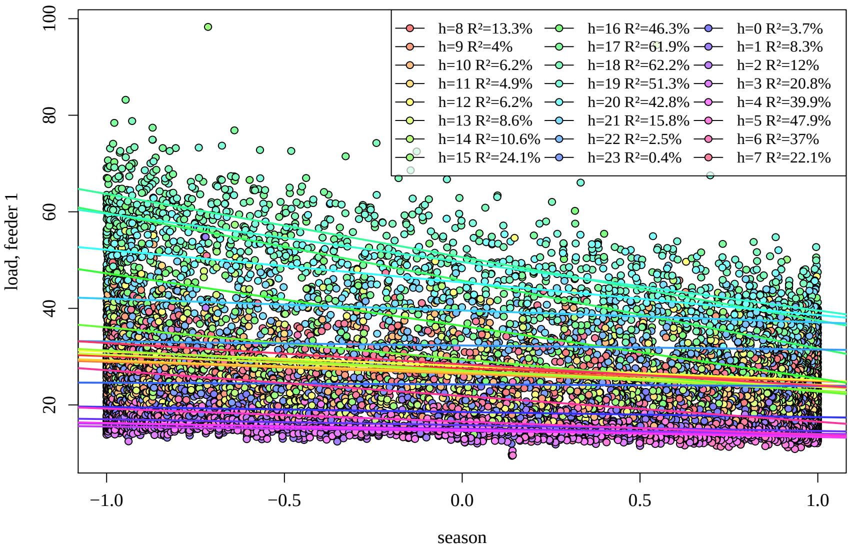


Figure : Linear dependence of `season` and `temperature`



R^2 = 5.6%
linear trend
load, feeder 1
season
100
80
60
40
20
−1.0
−0.5
0.0
0.5
1.0


h=8 R²=13.3%
h=9 R²=4%
h=10 R²=6.2%
h=11 R²=4.9%
h=12 R²=6.2%
h=13 R²=8.6%
h=14 R²=10.6%
h=15 R²=24.1%
h=16 R²=46.3%
h=17 R²=61.9%
h=18 R²=62.2%
h=19 R²=51.3%
h=20 R²=42.8%
h=21 R²=15.8%
h=22 R²=2.5%
h=23 R²=0.4%
h=0 R²=3.7%
h=1 R²=8.3%
h=2 R²=12%
h=3 R²=20.8%
h=4 R²=39.9%
h=5 R²=47.9%
h=6 R²=37%
h=7 R²=22.1%
load, feeder 1
100
80
60
40
20
−1.0
−0.5
0.0
0.5
1.0
season

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Temperature analysis for feeder 1:

$$\text{load}_t = \beta_0 + \beta_1 \text{season}_t + \varepsilon_t$$

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  28.1085     0.1266  222.04   <2e-16 ***
season       -4.6424     0.1790  -25.94   <2e-16 ***
```

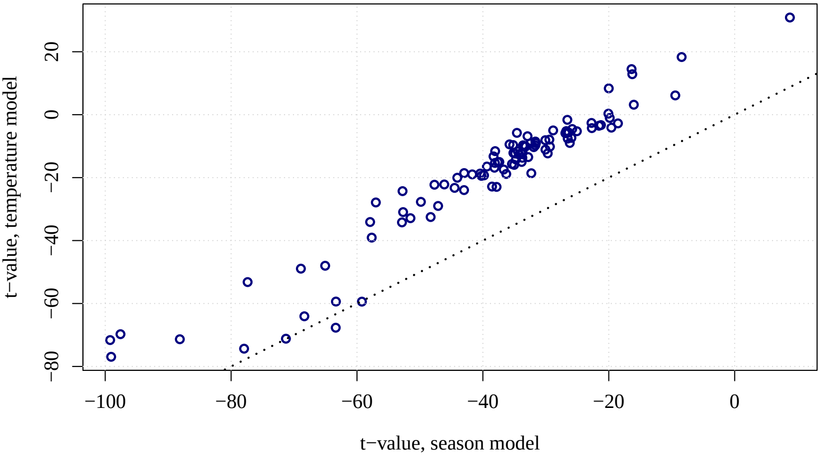
$$\text{load}_t = \beta_0 + \beta_1 \text{temperature}_t + \varepsilon_t$$

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 29.89992    0.27637 108.186   <2e-16 ***
temperature -0.17087    0.02321  -7.361    2e-13 ***
```

```
  if  normality and iid assumption holds:
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
    .    0.1          1
```

- t-value of season model much larger.
- p-values not reliable

T-values for season and temperature model for all feeders

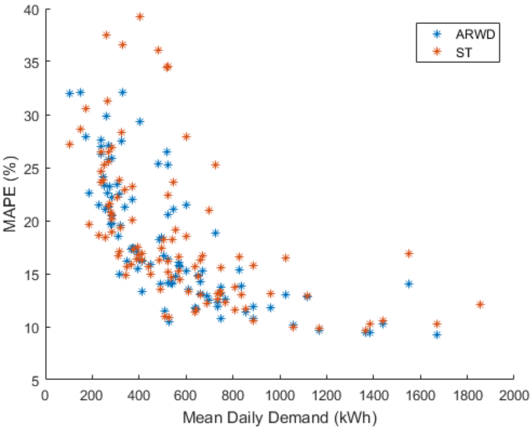


$$\text{load}_t = \beta_0 + \beta_1 \text{temperature}_t + \beta_1 \text{temperature}_t^2 + \beta_1 \text{temperature}_t^3 + \varepsilon_t$$

```
lm(formula = load ~ temperature + I(temperature^2) + I(
    temperature^3))
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)       29.0085173  0.4128066  70.271  < 2e-16 ***
temperature        0.3491895  0.1256890   2.778  0.00548 **
I(temperature^2) -0.0575065  0.0122314  -4.702 2.62e-06 ***
I(temperature^3)  0.0016600  0.0003495   4.750 2.07e-06 ***
```

- small p-values for all temperature effects

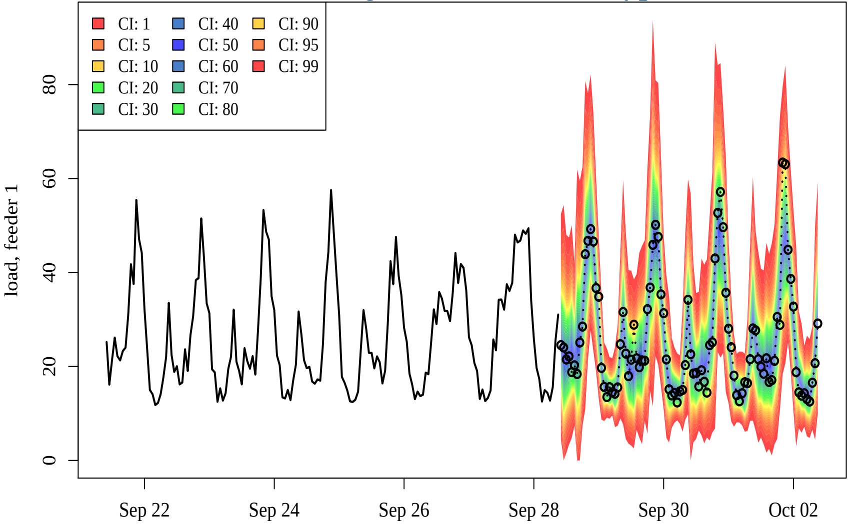
Errors and substation demand



Summary

- Short term forecasts (up to 4 days ahead)
- Weather (Temperature/Wind Chill) minimal impact on the forecast accuracy
  $\rightarrow$ seasonal deterministic components more relevant
- Simple 4 week average a competitive benchmark
- Autoregressive model with seasonal model best forecasting accuracy
- Strong relationship between size of feeder and relative accuracy

Outlook: Probabilistic forecasting results (ARWDY type model)



Future

- Further development of probabilistic methods
- More sophisticated investigations into weather impact
- Investigating of public holidays and clock-change effects
- Investigate the aggregation relationship
- Rolling and real time forecasts