Mathematical model of quantum cognition applied to bistable perception

Introduction

Bistable perception (optical illusions) is a phenomenon where the brain perceives spontaneous and unpredictable sequence of changes to a given stimuli, a famous example of an optical illusion is the 'Rabbit-Duck' image. Here, the observer can perceive two states, either a rabbit or a duck and then will spontaneously switch between the two. This is an example of a stationary optical illusion, during our research we focused on a non-stationary optical illusion, a Lissajous-Curve. Some research suggests that one can model the 'flipping' of the bistable perception using the quantum Zeno effect, an effect utilising the constant observation of an ambiguous bistable stimuli, and in doing so we can derive a quantum probability model that attempts to explain human cognition and perception. This project aims to evaluate the current proposed model and to extend it with the objective of developing an improved model with the understanding and use of quantum probability theory and to design an experiment to test predictions of our improved model.

What is a Lissajous Curve?

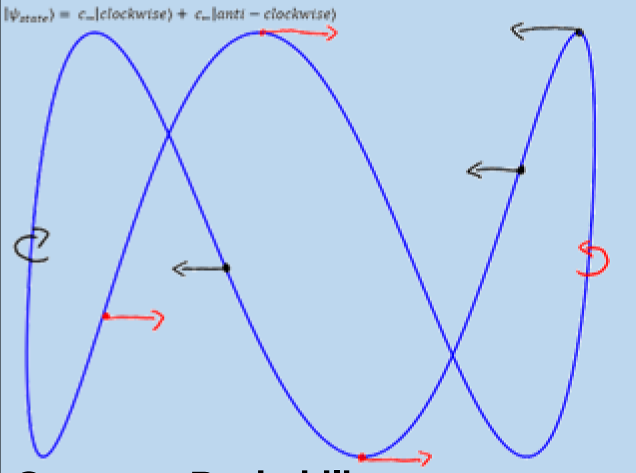
A Lissajous-Curve is a two-dimensional curve that are typically perceived as rotating in three-dimensional space, around the x-axis or the y-axis and they are formed from the intersection of two sine curves

$$x(t) = \sin(at + \delta)$$

$$y(t) = \sin(bt)$$

a/b are the lobe parameters (number of peaks) in the y/x direction (respectively). For simplicity, a will always be one so the parameter b alone determines the number of lobes. For the curve below $b = 3$.

δ is the phase difference, how far one wave is in front or behind the other, this parameter is responsible for the movement of the curve



Quantum Probability

In quantum physics, the results of any experiment is called a state and before observation they are said to be a superposition, mathematically shown below, which is a mix of all possible states it can be in. In our case, clockwise or anti-clockwise

$$|\psi\rangle = c_{\rightarrow}|\rightarrow\rangle + c_{\leftarrow}|\leftarrow\rangle$$

The probability of being in a given state (say clockwise) is given by

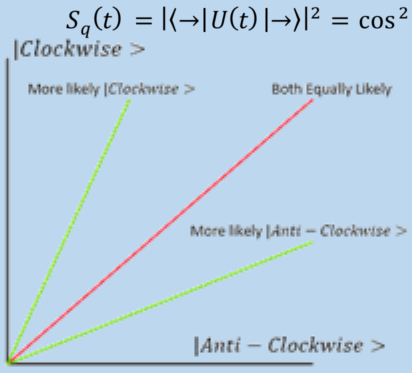
$$P_{\rightarrow} = |c_{\rightarrow}|^2$$

When the stimuli is not actively being watched, it decays at a rate related to the rotation matrix

$$U(t) = \begin{pmatrix} \cos\Gamma t & -i\sin\Gamma t \\ i\sin\Gamma t & \cos\Gamma t \end{pmatrix}$$

Where Γ = the decay rate. The quantum survival probability is then given by

$$S_q(t) = |\langle\rightarrow|U(t)|\rightarrow\rangle|^2 = \cos^2\Gamma t \approx 1 - \Gamma^2 t^2 + \cdots$$



This graph demonstrates quantum probability, the classical states are orthogonal to each other , i.e. they are the axes, and the superposition states are in between these two directions.

Quantum Zeno Effect

When we measure a quantum system, any superposition of states collapses into one of the classical outcomes, clockwise or anti-clockwise. After the measurement the quantum state begins to evolve again, i.e. the superposition of states begins to form again. The quantum Zeno effect states that if we perform these measurements frequently enough we can prevent the quantum state from evolving for extended periods of time, i.e. the superposition of states is unable to form. If we repeat this measurement N times spaced out by regular intervals Δt, such that $t = N\Delta t$ the quantum survival probability is found to be

$$S_q(N) = (1 - \Gamma^2\Delta t^2)^N \approx e^{-N\Gamma^2\Delta t^2}$$

$$S(t) \approx e^{-\gamma t};\ \gamma = \Gamma^2\Delta t$$

Even if t is arbitrarily large, we can see that if Δt is small we have that

$$S_q(t) \approx 1$$

An analogy of the quantum Zeno effect states that 'a watched quantum kettle would literally never boil.'

In the case of a quantum model of bistable perception, for Lissajous curves, the quantum Zeno effect would suggest that the stability of the perceived state increases with frequency.

The Necker-Zeno Model

The expected lifetime of a quantum state, or 'dwell-time' $< T >$, in this context is the average time you perceive a rotation direction, this can be calculated using a standard probability distribution function [1]

$$< T > = \int_0^\infty t\left[-\frac{dS(t)}{dt}\right]dt = \gamma\int_0^\infty te^{-\gamma t}dt = 1/\gamma$$

The term in the square brackets is the time derivative of the cumulative distribution function.

$$F(t) = P(t \geq T) = 1 - S(t)$$

Rewriting in terms of a 'lifetime' under no observation $t_0 = 2\pi\Gamma^{-1}$, we get that

$$< T > = {t_0^2}/{4\pi^2\Delta t}$$

Scan the above QR codes to view all source codes And Lissajous Curve videos

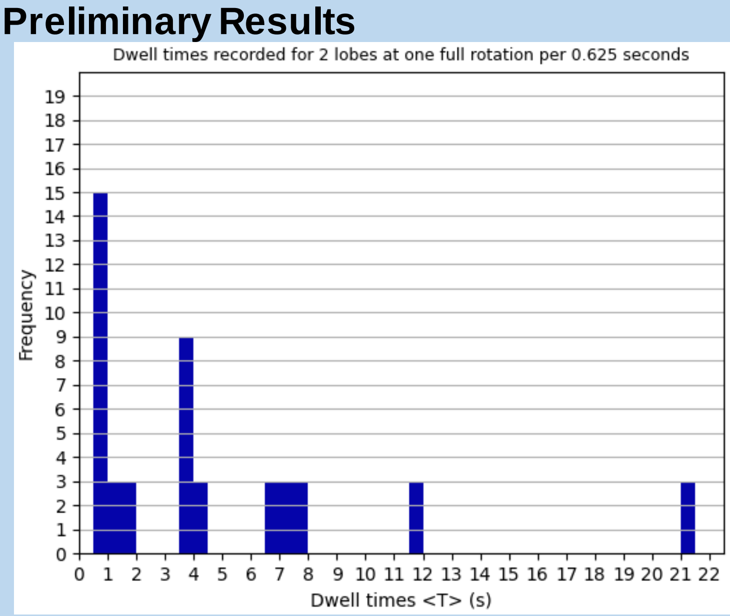
Revised Changes to the Model

- Δt is the time between the overlaps of the lines of the Lissajous curve, known as self-occlusions, rather than an observational update period as described in the literature [1]. This change was made due to data suggesting that the direction of rotation almost always flips during the self-occlusions. [2]
- A measurement will be the cognitive 'decision' of which direction a Lissajous curve moves rather than direct sensory observation, or input. This change was made as one can argue that the 3D perception of the Lissajous curve is not formed until later in the hierarchy of visual processing.

Proposed Experiment

- The curves corresponding to the parameters we wanted to study were prepared in advance of the experiment.
- The main experiment will be split into segments, 1 minute where the participant views the Lissajous curve and 20 seconds of ''off-time'. The parameters of the Lissajous curves to be tested are chosen randomly.
- When a segment starts, the participant will indicate the initial direction of the moving Lissajous curve using the mouse. Left-click for left (clockwise from above), Right-click for right (anti-clockwise from above) and the middle-button for an unclear direction. Any changes in perceived direction for the remainder of the block period was recorded using the same mouse-click method.
- The participants repeated the above process for each of the pre-prepared curves. The quantity being measured is the time in-between successive changes of perceived direction. Not necessarily mouse-clicks.

Preliminary Results



References

[1] - Atmanspacher, H. and Filk, T. (2013), The Necker–Zeno Model for Bistable Perception. Top Cogn Sci, 5: 800-817. https://doi.org/10.1111/tops.12044

[2] -Atmanspacher, et al.*"Cognitive Time Scales in a Necker-Zeno Model for Bistable Perception." The Open Cybernetics & Systemics Journal 2 (2008): 234-251.*

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