

Can misconceptions in physics pedagogy be modelled with quantum cognition?

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Abstract

Under the framework of conceptual change, misconceptions are viewed as incorrect ideas to be replaced. This perspective has many issues, the main one being that it fails to account for the context-dependent nature of reasoning. This project will follow the framework of representational pluralism, which suggests students can hold multiple, often conflicting representations constructed from smaller intuitive building blocks known as phenomenological primitives (p-prims). This framework allows us to use a quantum cognitive approach to model these representations probabilistically as superposed states of incompatible concept pairs. After reviewing the literature, we developed a model of quantum cognition representing concepts as vectors in a Hilbert space. This was applied to several common misconceptions using theoretical values for their projections and then considered contextually for different question phrasing. The model illustrates how misconceptions can emerge, coexist with 'correct' ideas, and shift depending on context.

Keywords: Misconceptions, Quantum Cognition, Representational Pluralism, Physics Pedagogy, Hilbert Space Models

Introduction

Misconceptions in physics present a persistent challenge to its pedagogy, because when a student holds a misconception, they rely on intuitive reasoning that conflicts with formalised scientific principles. These misconceptions often coexist, or are

superposed, with the accepted scientific principles, and their activation is context dependent. For example, a student's interpretation of a question may vary depending on how the information is presented, activating different p-prims. These context-dependent variations can in theory cause order effects and the conjunction fallacy, both of which cannot be modelled under classical probability.

The conjunction fallacy occurs when $P(A \wedge B) > P(A)$ or $P(A \wedge B) > P(B)$. Order effects describe how probability can depend on the order of events, for example if $P(A \rightarrow B) \neq P(B \rightarrow A)$. These effects explain why reasoning can be modelled naturally using quantum cognition. The aim of this project is to create a quantum cognition model representing a student's conceptual state as a superposition of concept pairs; one misconception and one accepted concept, and how it shifts depending on context.

Project Background

Physics is widely recognised for being a challenging area of study, which can discourage many students from pursuing it. Understanding how students' reason about physics concepts and modelling the factors that give rise to misconception is essential for improving its pedagogy. The traditional pedagogical approach in physics is conceptual change theory, which argues that students need to discard misconceptions and replace them with the scientifically accepted theory. (Posner et al., 1982) However, evidence shows that under this framework misconceptions are suppressed rather than replaced. Furthermore, it can be argued that these 'incorrect' ideas do not need to be replaced at all, and that this could cause ethical concerns on forcing belief change. (Booth and Paillusson, 2024)

Representational pluralism is the alternative to this, in which students can hold multiple, potentially conflicting representations simultaneously. From this perspective, students can apply scientifically accepted principles without discarding their intuitive reasoning, which ultimately encourages independent thinking and a hint of discovery in their learning. This area of research stood out to me because of the reluctance I originally had in choosing physics, feeling I wasn't capable. I believe this was influenced by the way physics was taught, through conceptual change rather than representational pluralism.

Investigating representational pluralism in physics pedagogy allows me to explore how learning can be better aligned with how students think. Developing a quantum cognitive model of misconceptions can formalise this understanding and potentially provide teachers with a better insight into students' reasoning.

Literature Review

Under traditional conceptual change approaches, misconceptions are seen as ideas that need to be replaced their scientifically correct counterparts. Research in physics education highlights that students' intuitive reasoning is context-dependent, and misconceptions tend to persist despite attempting to eradicate and replace them. For instance, Hammer (1996) observed that students could correctly articulate scientific principles in one context, but when problems were framed differently, they reverted to intuitive reasoning.

These studies suggest learners' misconceptions can coexist with accepted scientific principles, surfacing differently depending on the framing. For example, diSessa (1993) found that students often activate intuitive knowledge elements called 'p-prims' selectively depending on the context of the problem. Smith, diSessa and Roschelle (1993) showed that students' correct and incorrect conceptions can coexist, with reasoning shifting depending on how questions are presented. This is the justification behind representational pluralism, in which students' conflicting representations can be broken down into p-prims. These p-prims can be activated selectively dependent on context, which explains variability in student responses across differently framed questions.

Quantum cognition provides a natural framework for the context-dependent activation of p-prims. In quantum cognition, conceptual states are represented as vectors in a Hilbert space, allowing multiple conflicting representations to exist in superposition. (Busemeyer and Bruza, 2012) When a student considers a question, their state is projected onto the corresponding vector, and the squared magnitude of the projection gives the probability of selecting that answer, as shown below:

$$P(A) = ||P_A|\psi\rangle||^2$$

If the state is first projected onto A, the probability of then choosing B from this new state is:

$$P(A \rightarrow B) = ||P_B P_A |\psi\rangle||^2$$

A well-known example of the conjunction fallacy is the Linda problem, where participants judge 'Linda is a bank teller and is active in the feminist movement' as more probable than 'Linda is a bank teller' on its own. (Tversky and Kahneman, 1983) This can be interpreted using quantum cognition by looking at the features 'bank teller' and 'feminist' as incompatible vectors in a Hilbert space. The p-prims underlying are activated dependent on the context, which is a short brief on Linda given before the question is asked, and the sequence. Sequential projections of the conceptual state onto these vectors explain why the probabilities are non-additive.

Methodology

The aim of this project is to build a quantum cognition model of misconception in physics, using Hilbert space formalism to represent conceptual states probabilistically as superpositions of potentially incompatible concept pairs. Concept pairs were chosen from the Institute of Physics spark database of common misconceptions in physics education research.

An example used in the modelling was the following question:

Which is more likely:

A: Temperature change occurs because the kinetic energies of atoms move toward equilibrium.

B: Temperature change occurs because heat flows through substances.

C: Both A and B occur simultaneously.

These concepts were represented as vectors in a 2D Hilbert space using hypothetical figures:

$$|A\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |B\rangle = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

Where θ represents the degree of incompatibility between A and B, ranging from 0° , where the events would be fully compatible, to 90° , where the events would be mutually exclusive. Not A ($\neg A$) and not B ($\neg B$) would also be included as vectors on the Hilbert space, both orthogonal to their respective counterpart.

The initial cognitive state of the student is represented as

$$|\psi\rangle = \alpha|A\rangle + \beta|B\rangle$$

Where α and β represent the student's weighting of each concept before engaging in the question.

Projectors P_A and P_B are defined as the outer products:

$$P_A = |A\rangle\langle A|, \quad P_B = |B\rangle\langle B|$$

The probability of a student selecting A is given by the projection of the cognitive state onto that concept:

$$P(A) = |\langle A|\psi\rangle|^2 = ||P_A|\psi\rangle||^2 = \langle \psi | P_A | \psi \rangle$$

And would be represented similarly for concept B. For sequential considerations the probability of selecting A then B is represented as:

$$P(A \rightarrow B) = ||P_B P_A |\psi\rangle||^2 = \langle \psi | P_A P_B P_A | \psi \rangle$$

And the probability of selecting B then A as:

$$P(B \rightarrow A) = ||P_A P_B |\psi\rangle||^2 = \langle \psi | P_B P_A P_B | \psi \rangle$$

Where, even though they reach the same result, the probabilities $P(A \rightarrow B)$ and $P(B \rightarrow A)$ will not always be equal.

We will be looking at the case of $P(A \rightarrow B)$ for the conjunction fallacy, as this represents the probability that the student first considers the correct concept A, and then the misconception B.

Results

We modelled hypothetical student responses to three physics concepts using a two-dimensional quantum cognitive framework. The two concepts chosen were heat transfer and electric current. They were formatted as individual questions, each asking which outcome would be more likely between A, B and A and B. The relevant concepts were represented by vectors A (accepted scientific concept), and B (secondary concept, a misconception). Probabilities were calculated using the standard methods mentioned in the methodology.

A conjunction fallacy occurs when $P(A \rightarrow B) > P(B)$. Sequential reasoning can increase the likelihood of the conjunction, even when the secondary concept is unlikely to occur from the initial state.

To interpret the results, each question is examined to show how the alignment of the student state vector with the concept vectors influences the probabilities and affects the occurrence of conjunction fallacies.

In all examples, concepts A and B are moderately incompatible, with $\theta=25^\circ$ and in the first example, the student's initial cognitive state is biased towards A.

Scenario: An ice cube is placed on a hot metal block

Which is more likely?

A: Temperature change occurs because the kinetic energies of atoms move toward equilibrium.

B: Temperature change occurs because heat flows through substances.

C: Both A and B occur simultaneously.

With vectors:

$$|\psi\rangle = \begin{pmatrix} 0.977 \\ 0.206 \end{pmatrix}, \quad |A\rangle = \begin{pmatrix} 0.600 \\ 0.800 \end{pmatrix}, \quad |B\rangle = \begin{pmatrix} 0.200 \\ 0.980 \end{pmatrix}$$

Probabilities:

$$P(A) = 0.565, \quad P(B) = 0.158, \quad P(A \rightarrow B) = 0.462$$

These values mean that while event B occurring from the initial state is unlikely, sequential reasoning through A increases the perceived likelihood of B being true. In terms of p-prims, activating the correct concept A primes the misconception B, leading to the conclusion of both being judged as true.

The next example models a scenario where the student's initial cognitive state is balanced between A and $\neg A$.

Scenario: A battery is connected to a small light bulb in a simple circuit.
Which is more likely?

- A: Electric current flows continuously through the circuit (correct).
- B: Current is used up as it flows through the circuit (common misconception).
- C: Both A and B occur simultaneously.

With vectors:

$$|\psi\rangle = \begin{pmatrix} 0.164 \\ 0.987 \end{pmatrix}, \quad |A\rangle = \begin{pmatrix} 0.600 \\ 0.800 \end{pmatrix}, \quad |B\rangle = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

Probabilities:

$$P(A) = 0.788, \quad P(B) = 0.662, \quad P(A \rightarrow B) = 0.772$$

The results show that $P(B)$ is high, since the student's initial state is a mix between A and $\neg A$, but it still appears that $P(A \rightarrow B)$ is higher, meaning that the conjunction fallacy can still occur for higher probabilities of B.

Example	ψ (initial state)	θ (degrees)	P(A)	P(B)	$P(A \rightarrow B)$	Conjunction Fallacy?
Heat Transfer	$\begin{pmatrix} 0.977 \\ 0.206 \end{pmatrix}$	25°	0.565	0.158	0.462	Yes
Electric Current	$\begin{pmatrix} 0.164 \\ 0.987 \end{pmatrix}$	25°	0.788	0.662	0.772	Yes

Table 1. Probabilities for each question, including the student state vector ψ , primary concept vector A, secondary concept vector B, sequential projection $P(A \rightarrow B)$, and the presence of a conjunction fallacy.

The final stage of research was to compare the model with existing data and modify based on this, but unfortunately no empirical student data could be found. Therefore, all numerical values are hypothetical, included only to illustrate how the Hilbert space model may look.

UROS Experience

The UROS project has been an invaluable experience to me. Engaging with research independently to any curriculum gave me the freedom to research and explore my ideas. This specific area of research connected my interest in physics and philosophy, and I found it interesting to learn about the philosophical motivations behind the cognitive science supporting pedagogical methods in physics. It also provided me some good insight into the broad range of topics supported by physics, showing how different disciplines can overlap

This project challenged me to engage with difficult material in ways that I hadn't previously in taught modules. Reviewing literature, picking concepts and considering how they relate to my project improved my problem-solving skills. Furthermore, it showed me the difference in progression speed of research as opposed to normal academic work, which took persistence. In addition, I gained confidence in my ability to communicate complex and abstract ideas, both through discussion, and through planning of the poster presentation.

Conclusion

This project has demonstrated that quantum cognition provides a natural framework for modelling reasoning patterns behind some misconceptions in physics pedagogy. Representational pluralism gives the foundation for treating misconceptions as alternative conceptions that can exist simultaneously with established scientific understanding. Using Hilbert space formalism captures the context dependence of misconception, offering a way to represent the flexibility of student reasoning and the issues that can arise. While the model developed here is theoretical and only limited to 2 dimensions, it highlights promising possible directions for future work. Empirical testing with survey data from a range of participants would be important for further developing the model, and extending the model into higher-dimensional Hilbert spaces could better capture the complexity of reasoning across multiple conceptual contexts.

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