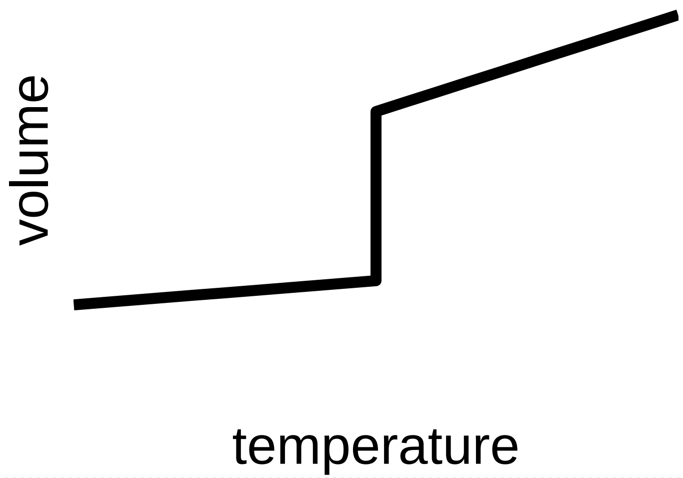
Correlated Electron Systems

4) Quantum Critical Points and non-Fermi Liquids

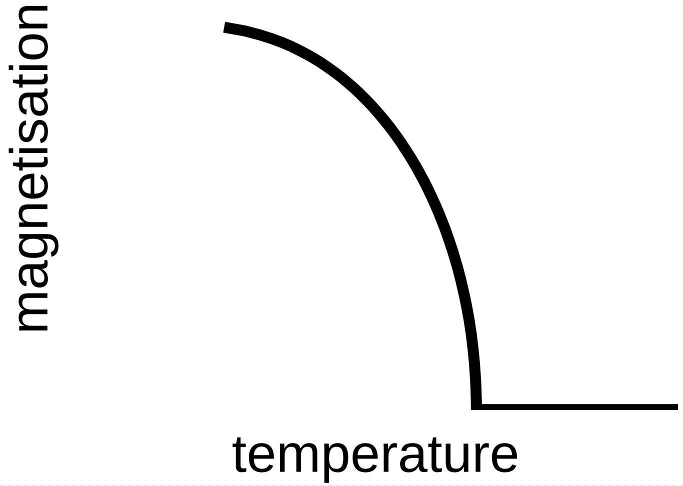
Löhneysen, H. v, et al. *Rev. Mod. Phys.*, *79*, 1015.

Classical Phase Transition

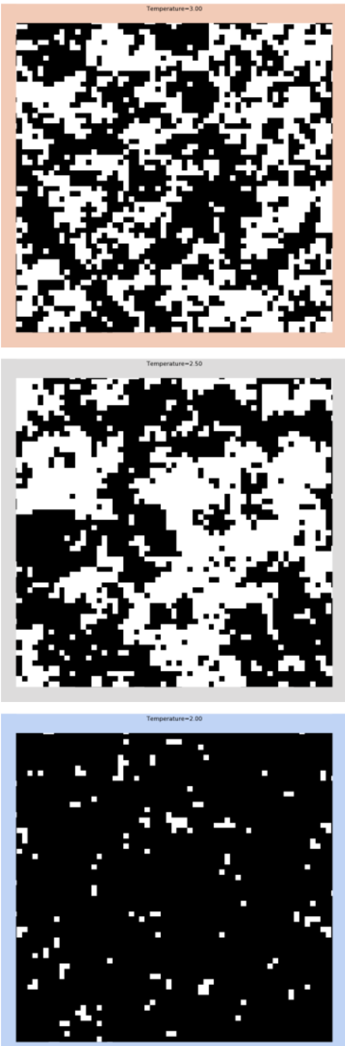
- 1st order – discontinuous
- Latent heat
- Different states
- Different volume
- Different entropy



- 2nd order – continuous
- No latent heat
- States identical at T_c
- Same volume
- Same entropy

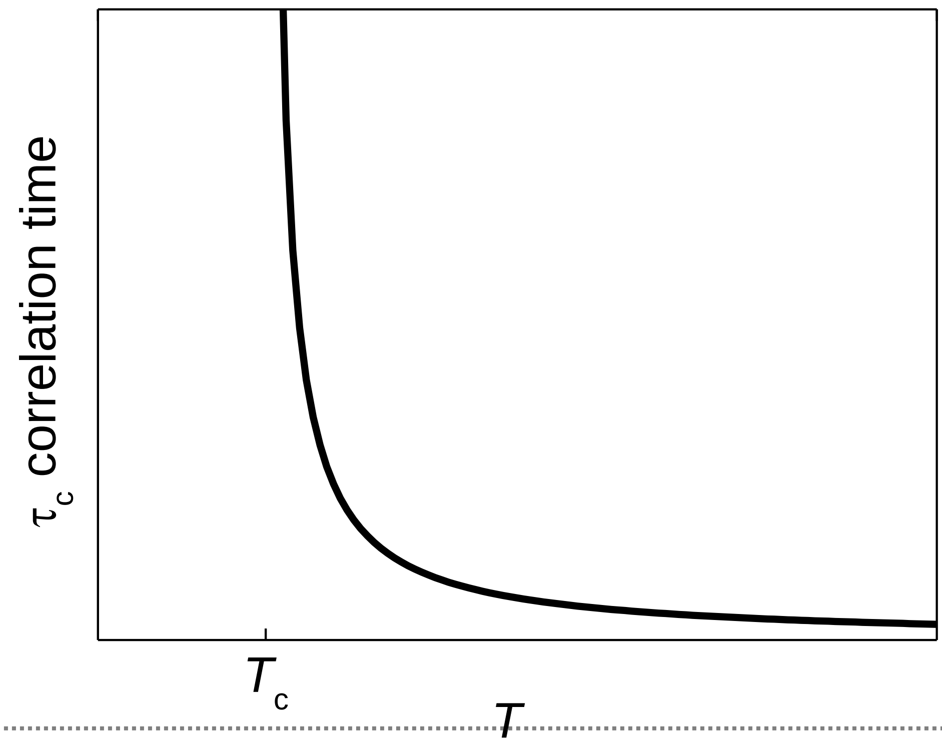


2nd order CLASSICAL phase transition



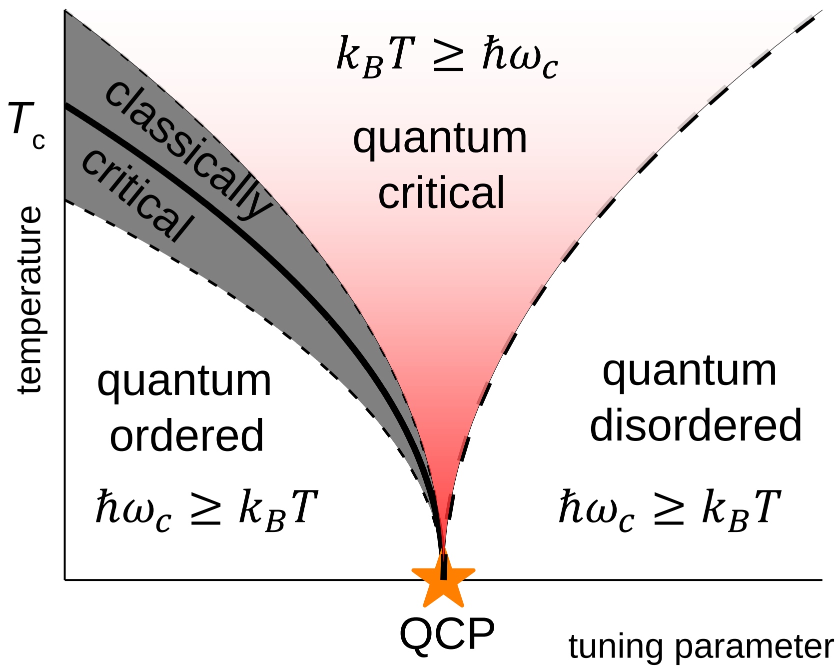
Phase transition at finite temperature

- Diverging correlation time
- quantum fluctuations $\hbar\omega_c \propto |t|^{\nu z}$
- classical fluctuations $k_B T_c$
- classical fluctuations dominate close to T



Quantum Critical Point (QCP)

Suppressing 2nd order phase transition

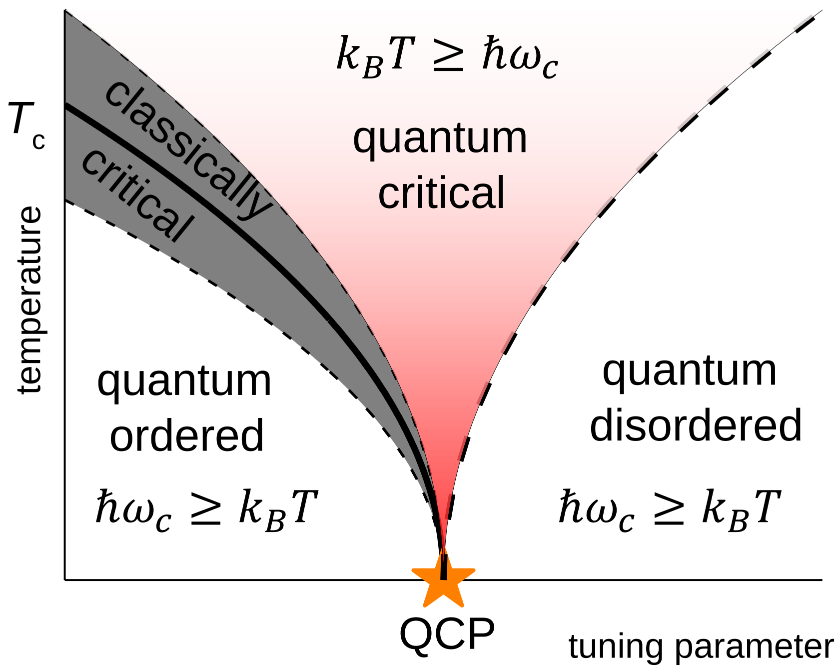


QCP at T=0

- 2nd order phase transition at zero temperature
- No thermal fluctuations $k_B T = 0$
- Quantum fluctuations down to lowest energies $\hbar\omega_c \geq 0$
- Drive phase transition
- Dominate finite temperature physics
- classical critical in $t < T_c^{\frac{1}{\nu z}}$

Quantum Critical Point (QCP)

Suppressing 2nd order phase transition



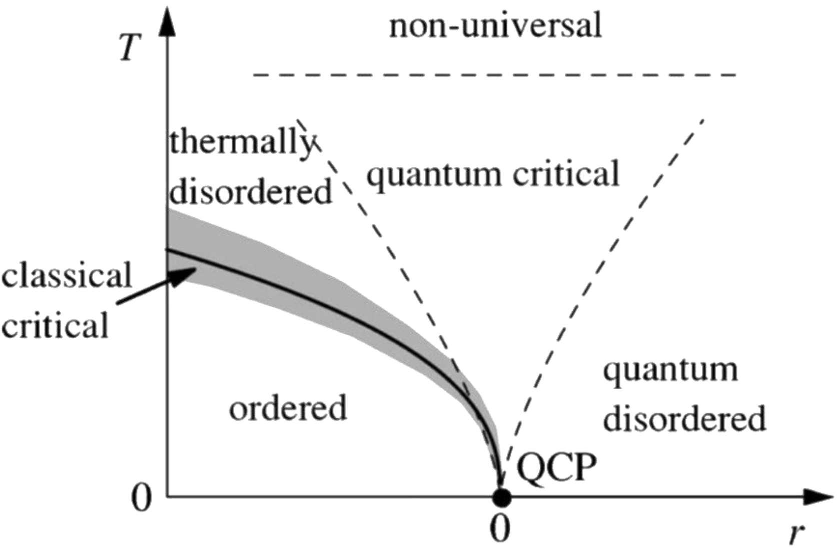
Quantum Criticality T > 0

- thermal fluctuations dominate
- mix of quantum ground states
- quantum and thermal excitations
- new behaviour with

$$d_{eff} = d + z$$

- quantum critical divergences

Conventional QCP



	$d=2$, $z=2$	$d=3$, $z=2$	$d=2$, $z=3$	$d=3$, $z=3$
$\alpha_{cr}\sim$	Tr^{-1}	$Tr^{-1/2}$	$Tr^{-3/2}$	Tr^{-1}
$C_{cr}\sim$	$T\ln\frac{1}{r}$	$-Tr^{1/2}$	$Tr^{-1/2}$	$T\ln\frac{1}{r}$
$\Gamma_{r,cr}=$	$\left(r\ln\frac{1}{r}\right)^{-1}$	$-(2r)^{-1}$	$(2r)^{-1}$	$\left(r\ln\frac{1}{r}\right)^{-1}$

(a)

	AFM, $z=2$, $d=3$	AFM, $z=2$, $d=2$	FM, $z=3$, $d=3$	FM, $z=3$, $d=2$
C/T	$\gamma-a\sqrt{T}$	$c\log(T_0/T)$	$c\log(T_0/T)$	$T^{-1/3}$
$\Delta\chi$	$T^{3/2}$	χ_0-dT		
$\Delta\rho$	$T^{3/2}$	T	T	
$T_{N/C}$	$(\delta_c-\delta)^{2/3}$	$(\delta_c-\delta)$	$(\delta_c-\delta)^{3/4}$	$(\delta_c-\delta)$
T_{I}	$(\delta-\delta_c)$	$(\delta-\delta_c)$	$(\delta-\delta_c)^{3/2}$	$(\delta-\delta_c)^{3/2}$
T_{II}	$(\delta-\delta_c)^{2/3}$	$(\delta-\delta_c)$	$(\delta-\delta_c)^{3/4}$	$(\delta-\delta_c)$

(b)

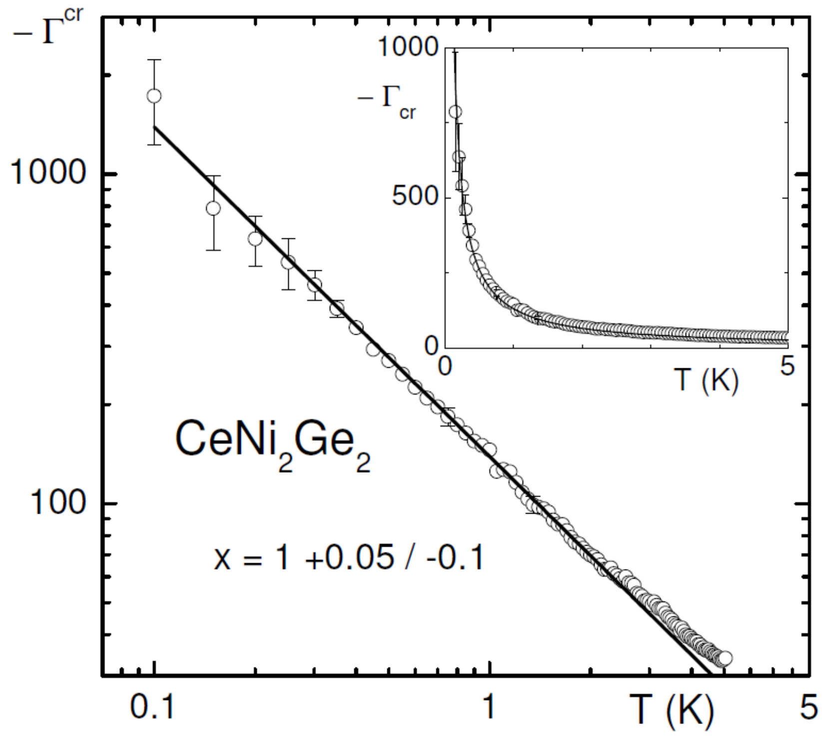
	Ferro, 3-dim	Ferro, 2-dim	AFM, 3-dim	AFM, 2-dim.
C_m/T	$-\log T$	$T^{-1/3}$	$\gamma_0-aT^{1/2}$	$-\log T$
χ_Q	$T^{-4/3}$	$-T^{-1}/\log T$	$T^{-3/2}$	$-(\log T)/T$
$\Delta\rho$	$T^{5/3}$	$T^{4/3}$	$T^{3/2}$	T

(c)

	Ferro, 3-d ($d=z=3$)	Ferro, 2-d ($d=2;z=3$)	Antiferr, 3-d ($d=3;z=2$)
C/T	$-\log T$	$T^{-1/3}$	$\gamma+\sqrt{T}$
$\Delta\chi$	$T^{-4/3}$	T^{-1}	$T^{-3/2}$
ρ	$T^{5/3}$	$T^{4/3}$	$T^{3/2}$

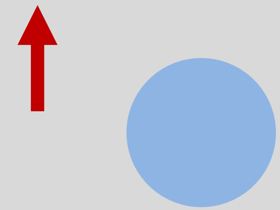
QCP Smoking guns

- Gruneisen ratio
- $\Gamma = \frac{\beta}{C_p} \propto \frac{\partial S/\partial p|_T}{\partial S/\partial pT|_p} \propto \frac{1}{E^*}\frac{\partial E^*}{\partial p}$
- Diverges as $E^* \to 0$
- Only at phase transitions

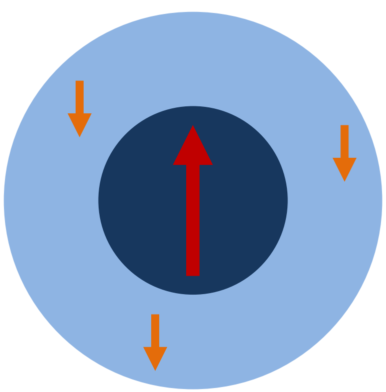


QCPs in Heavy Fermions

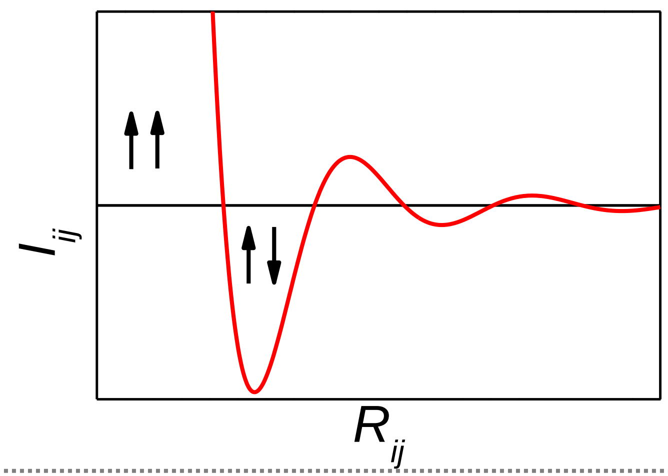
Ingredients
- Local moments (f-electrons) on periodic lattice
- Conduction electrons (s,p,d)



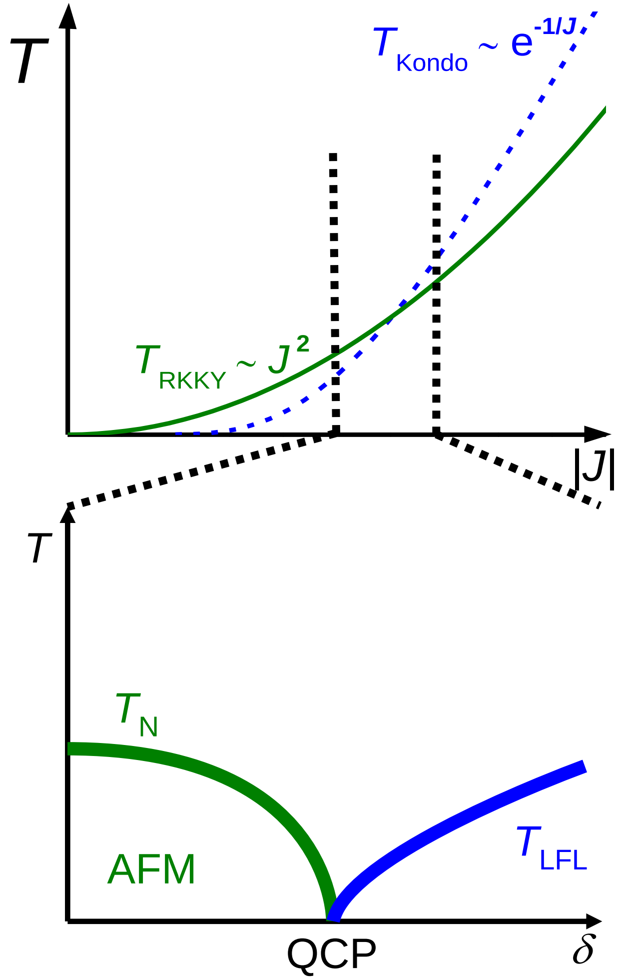
Kondo effect
- $H_K \propto J\,\vec{S}\cdot\vec{s}$
- Heavy quasiparticles
- Paramagnetic
- Fermi liquid



RKKY Interaction
- $H_{RKKY} \propto -I_{ij}\overrightarrow{S_i}\cdot\overrightarrow{S_j}$
- Magnetic ground state



Competing ground states



Doniach, Physica B+C 91, 231-234 (1977)

Exchange Interaction J between conduction and f-electrons

Weak J	RKKY dominates **AFM** ground state
crit. J_c	continous phase transition at T=0: **QCP**
Strong J	Kondo effect: paramagnetic **Heavy Fermi liquid**

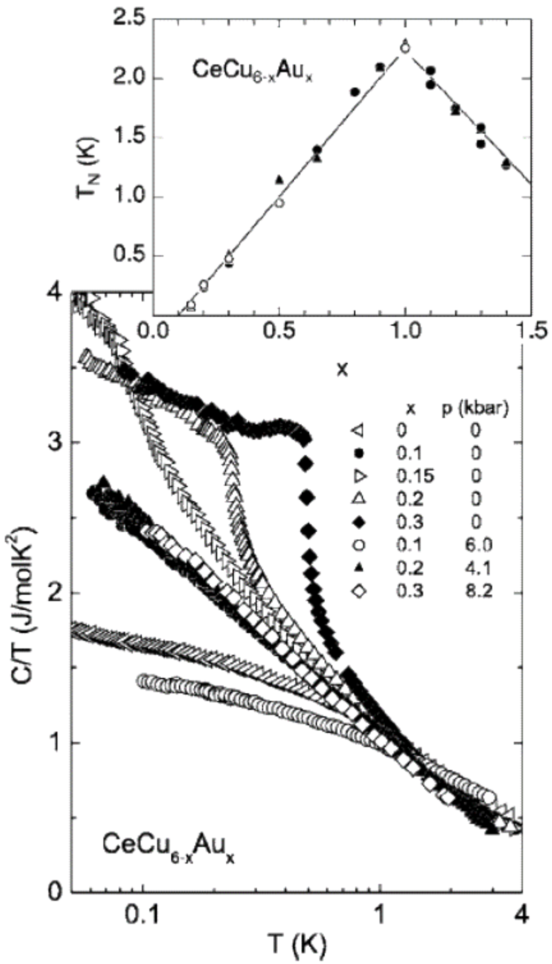
QCP experiments

$CeCu_{6-x}Au_x$

- Magnetism suppressed by Au substitution
- Divergent specific heat coefficient at QCP

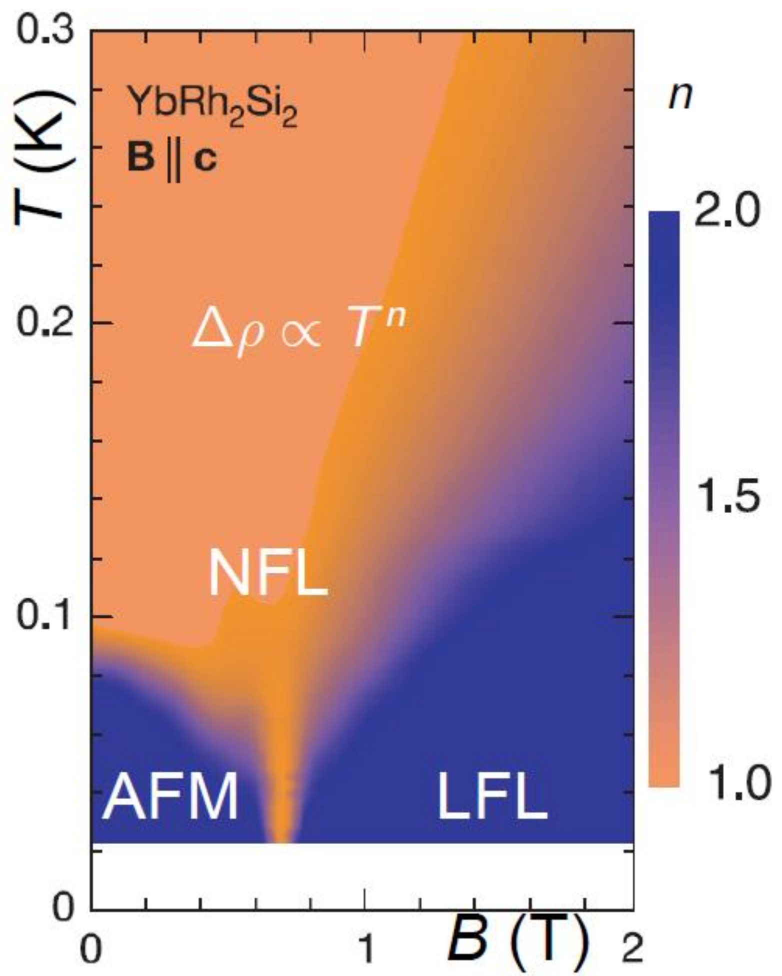
$$\frac{C}{T} \propto \ln\left(\frac{T_0}{T}\right)$$

- Seems to fit nicely but...
- scaling beyond order parameter scenario



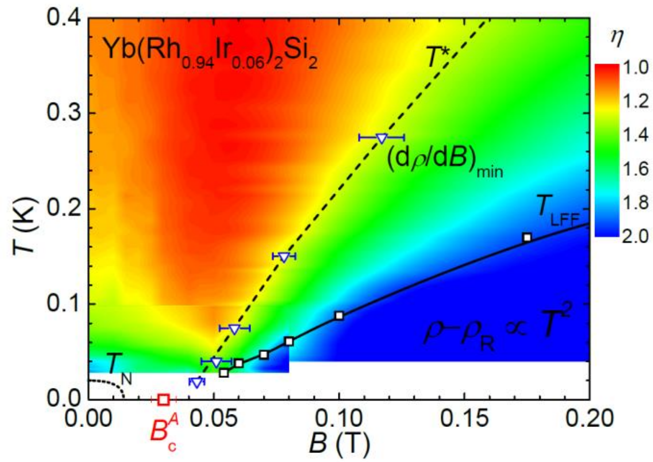
QCP experiments

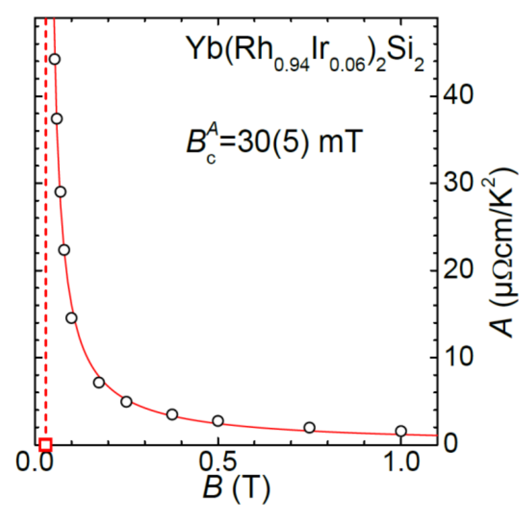
- $YbRh_2Si_2$
- Non-Fermi liquid behaviour
  - Susceptibility
  - Specific heat
  - Resistivity
- Singular at QCP



Divergent quasiparticle mass at QCP

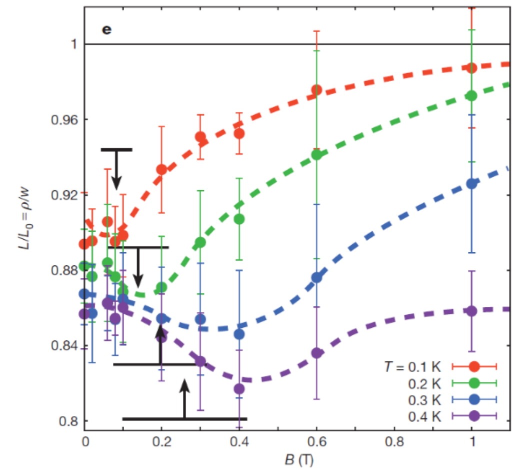
- Mass divergence smoking gun for QCP
- Vanishing Fermi liquid temperature
- $YbRh_2Si_2$

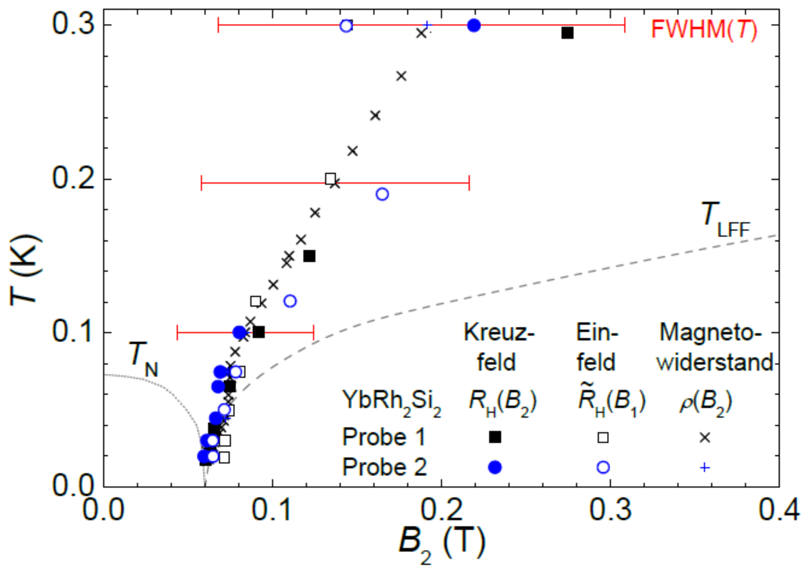




Beyond Order Parameter

- Magnetic transition
- Accompagnied by Fermi Surface collapse
- Hall effect in $YbRh_2Si_2$
- Violation of Wiedemann-Franz
- Breakdown of Quasiparticle concept





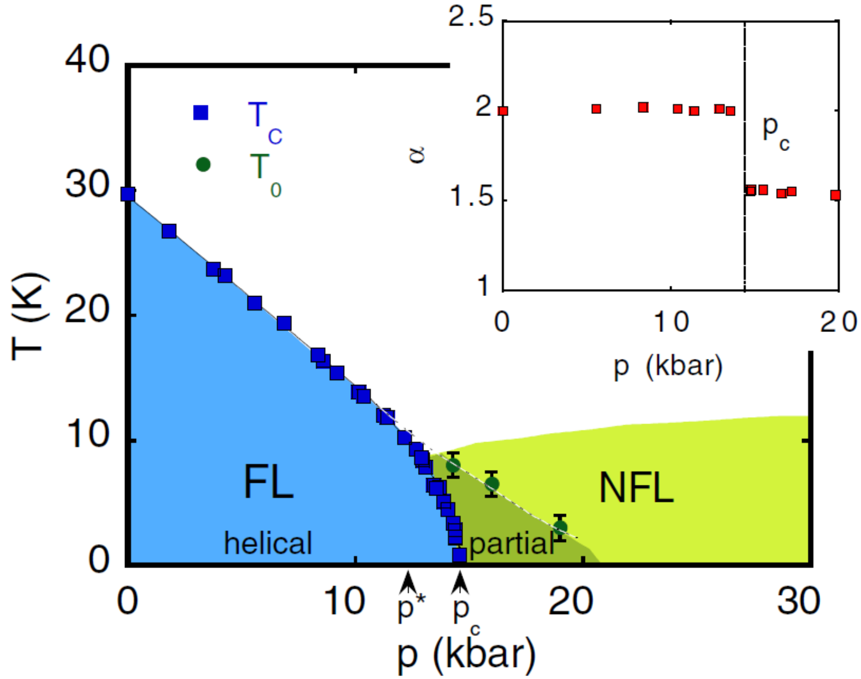
Friedemann, S, et al *PNAS*, *107*, 145477
Pfau, H. Et al. Nature 2012

nFL phases

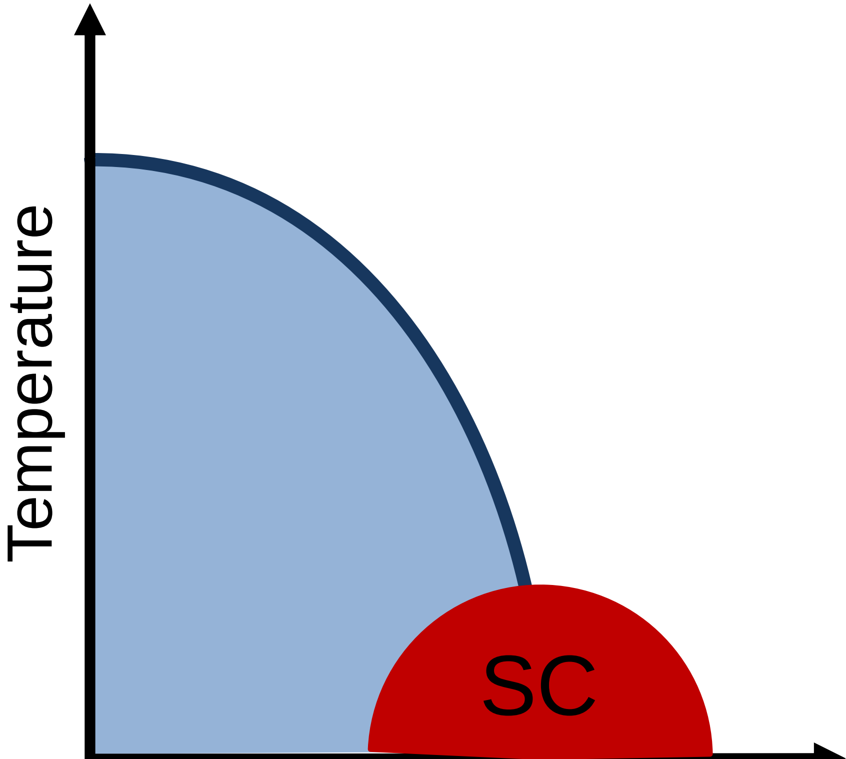
- MnSi -- many puzzles
- Compare $ZrZn_2$
- Non-Fermi liquid in resistivity

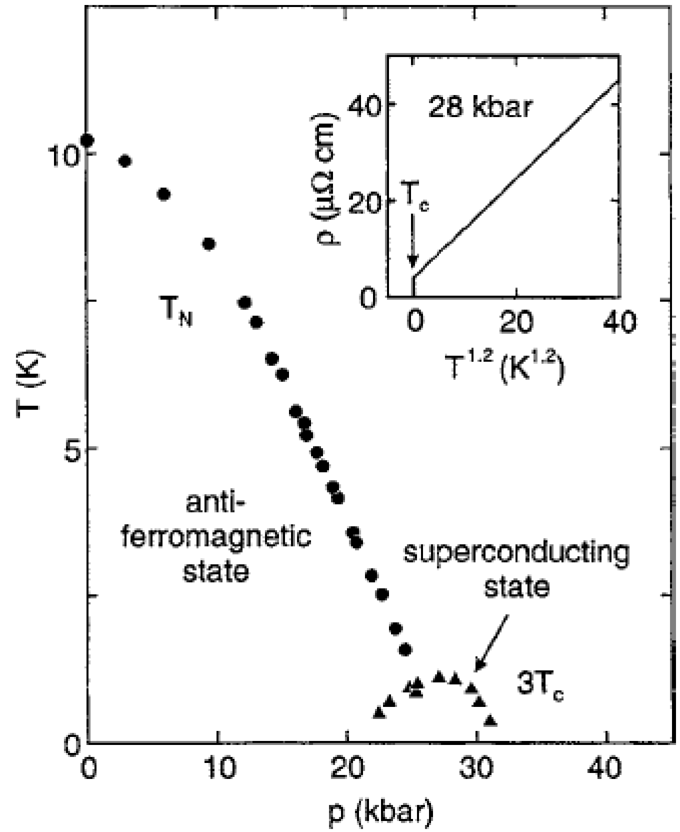
$$\rho - \rho_0 \propto T^{3/2}$$

- This exponent not understood
- Extended pressure range not understood



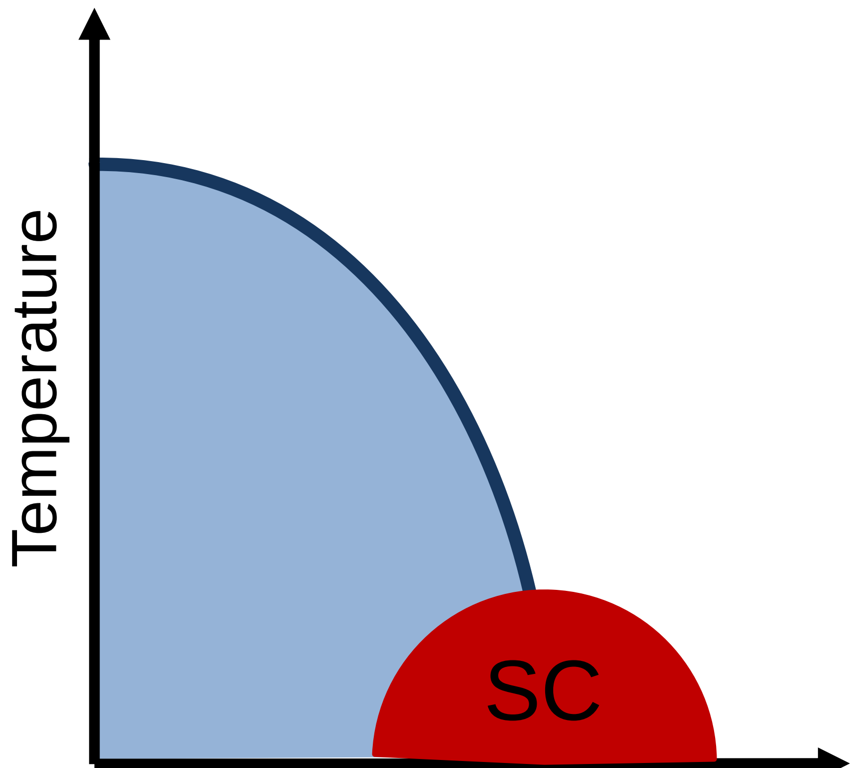
Unconventional Superconductivity at QCP

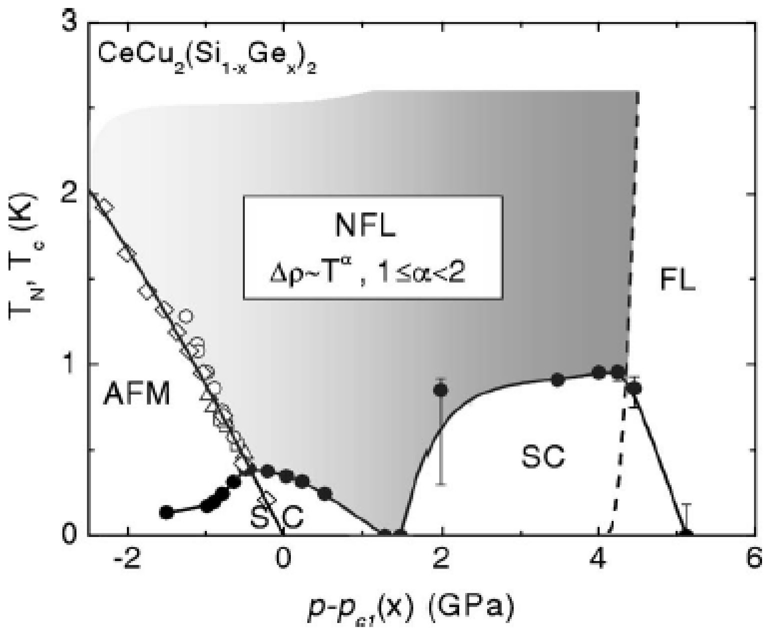




N. D. Mathur et al Nature 394, 39 (1998)

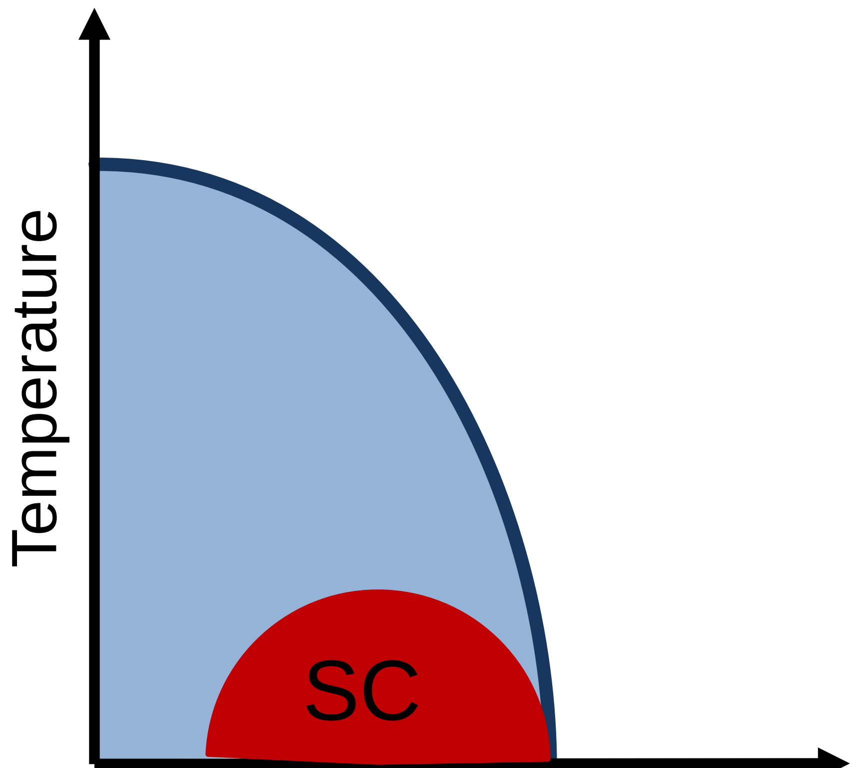
Unconventional Superconductivity at QCP

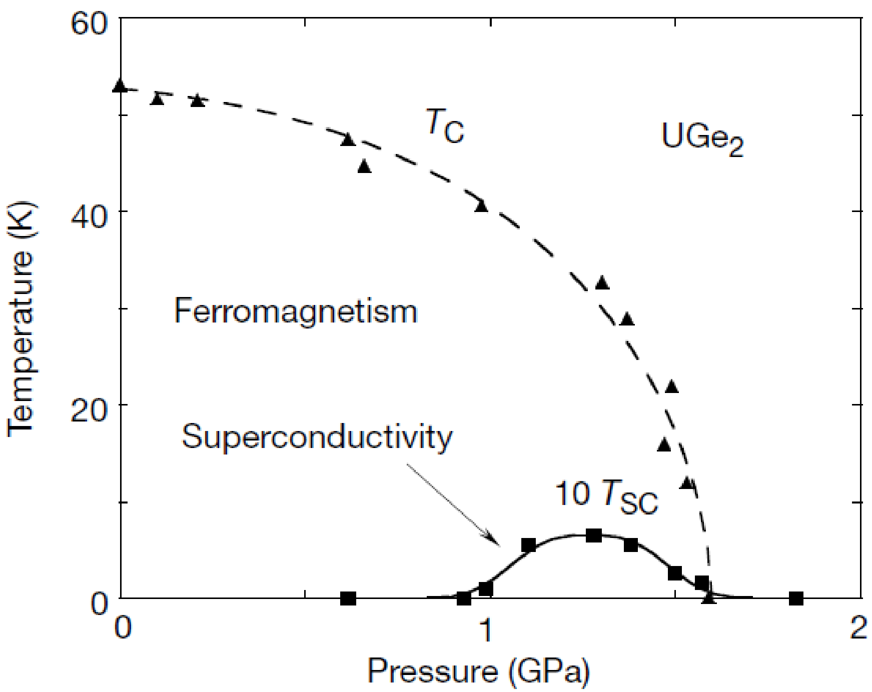




Yuan et al 2006

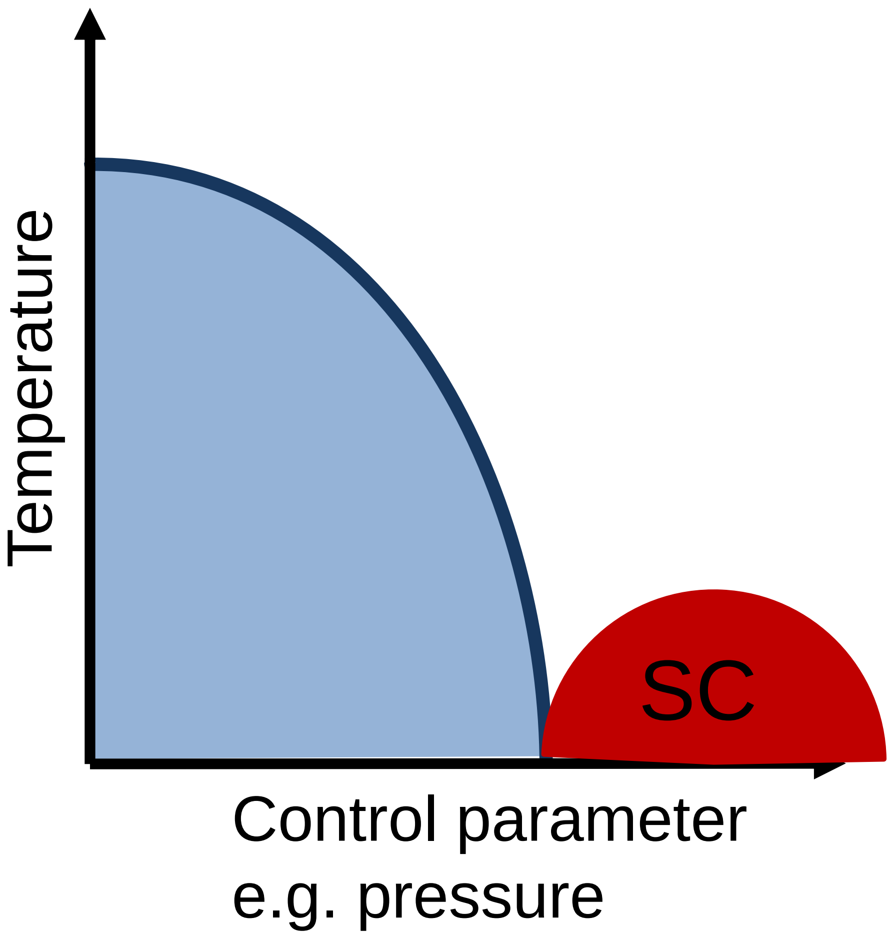
Unconventional Superconductivity at QCP

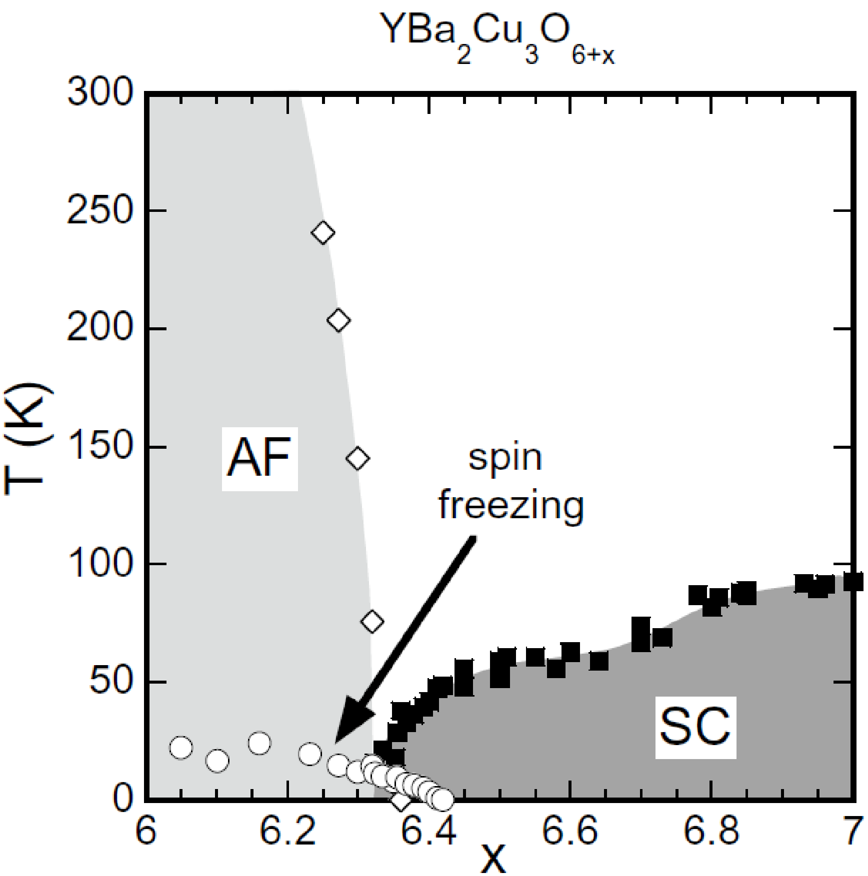




S. S. Saxena et al Nature 406, 587 (2000)

Unconventional Superconductivity at QCP

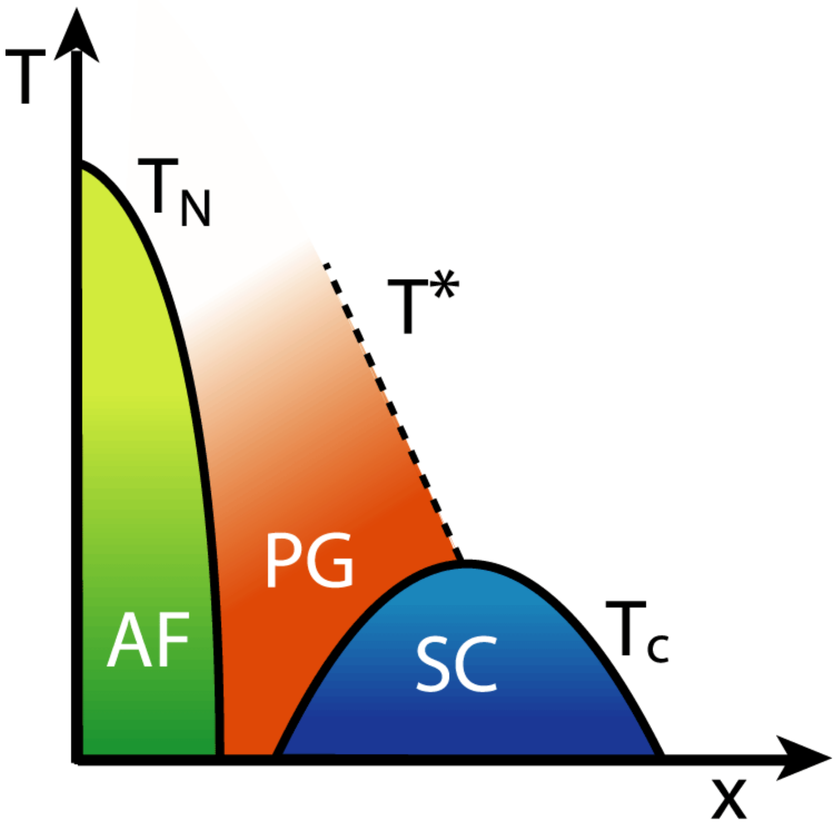




High Temperature Superconductors

Important issues:

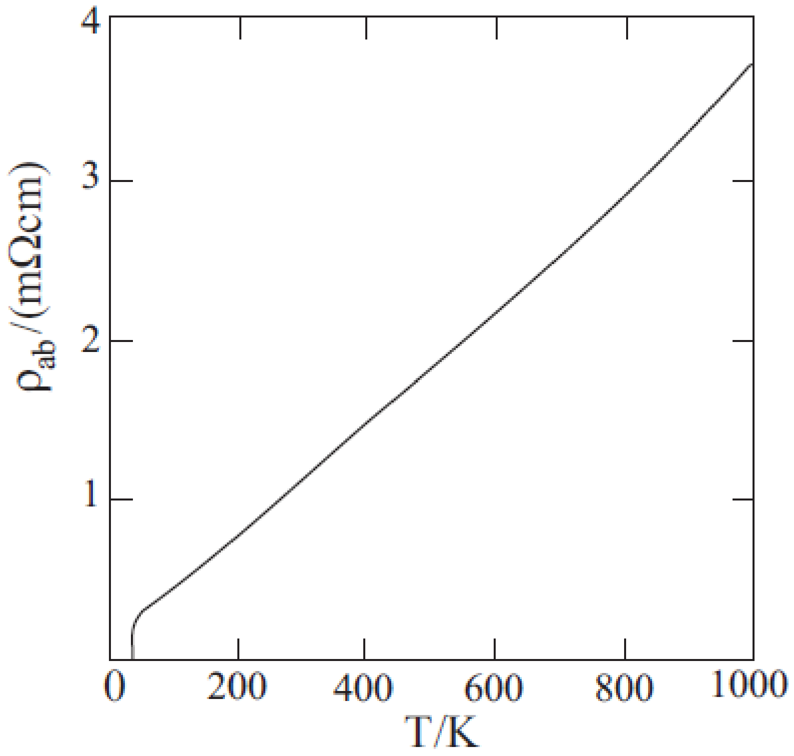
- Is this a "quantum critical" system?
- Where is the QCP point?
- Is the "pseudogap region" an ordered phase?
- Is there a competing ordered phase (not shown)?



nFL in Cuprates

- non-Fermi liquid in resistivity

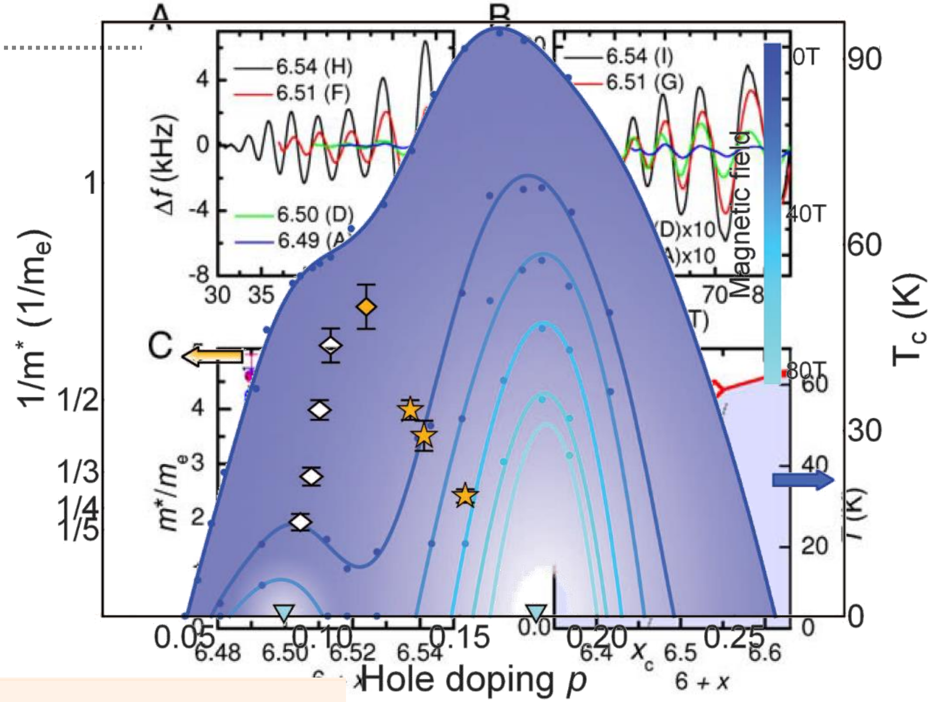
$$\rho \propto T$$

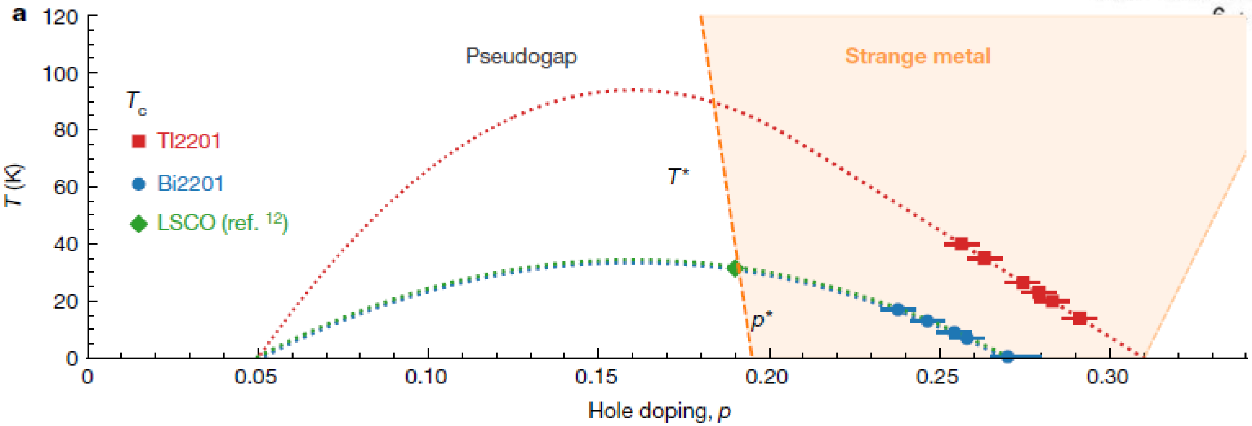


$La_{1-x}Sr_xCuO_4$
Takagi, H. et al. *PRL, 69*, 2975.

QCP(s) in Cuprates?

- Increase in quasiparticle mass
- Divergent form
- Smoking gun of QCP?
- Strange metal phase?





Sebastian 2010, Ramshaw 2015