Correlated Electron Systems

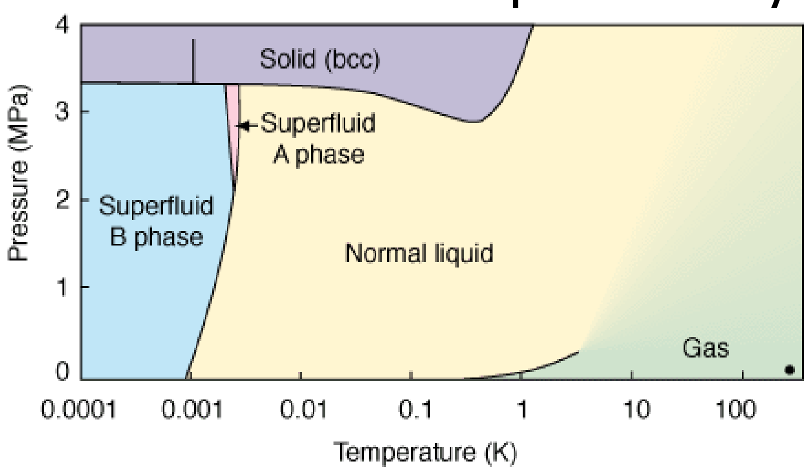
1) Fermi Liquid Theory

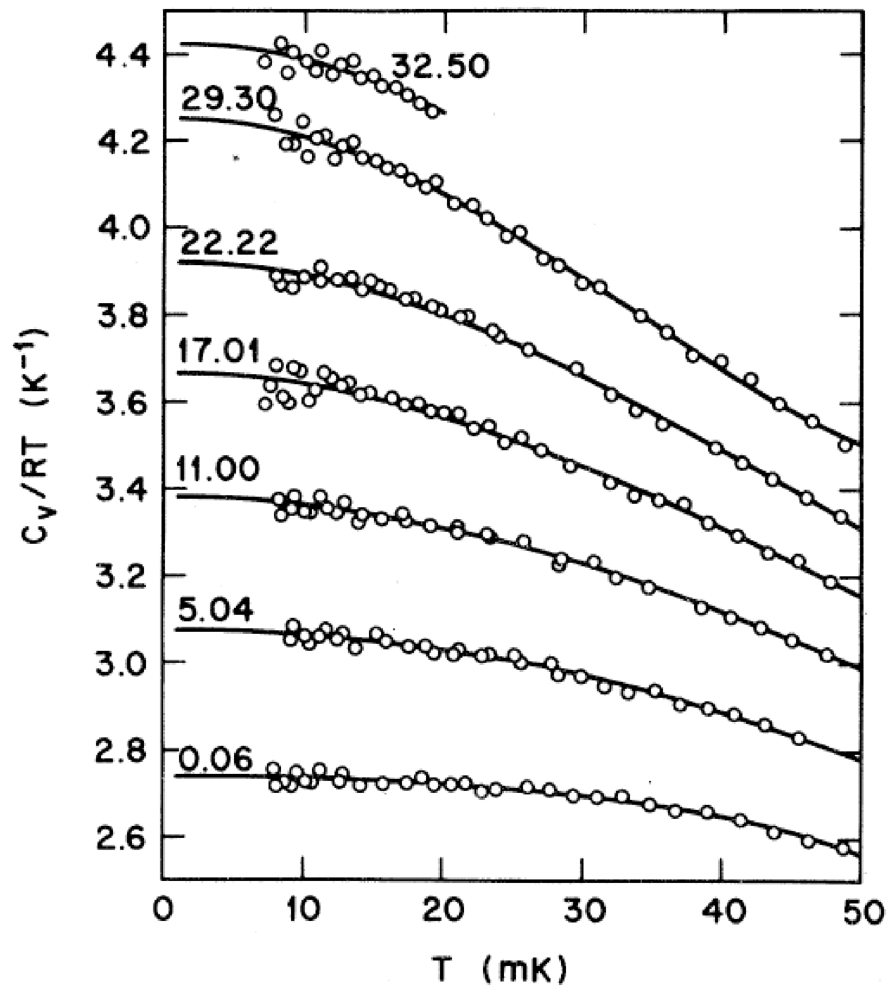
Literature

1. Coleman, P. *Introduction to Many-Body Physics*.
2. Pines and Nozieres: *The Theory of Quantum Liquids*

^{3}He

- Remains liquid (p=0)
- Fermions
- $\frac{C}{T} \propto g(E_F) \propto n^{1/3}$
- Stronger then expected increase under pressure
- Solution: Fermi liquid theory





Greywall 1983

3He Sommerfeld Wilson Ratio

1) Classical system: Curie Law

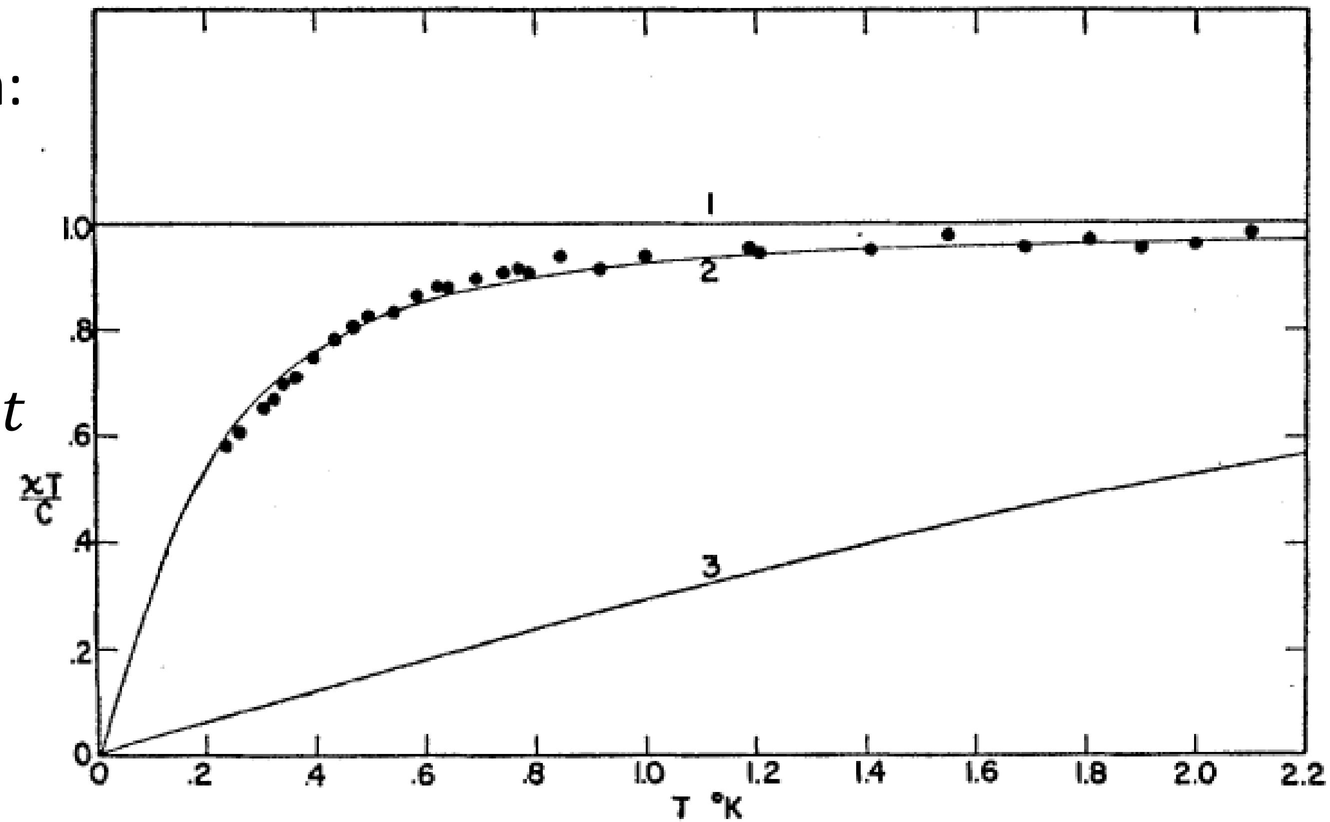
$$\chi = \frac{C}{T}$$

$$SWR = \frac{\chi_0}{\gamma_0} = const$$

3)

$$T_F = \frac{\hbar^2}{2m_0}(3\pi^2 n)^{2/3} = 5K$$

2) $T_F = 0.45K$



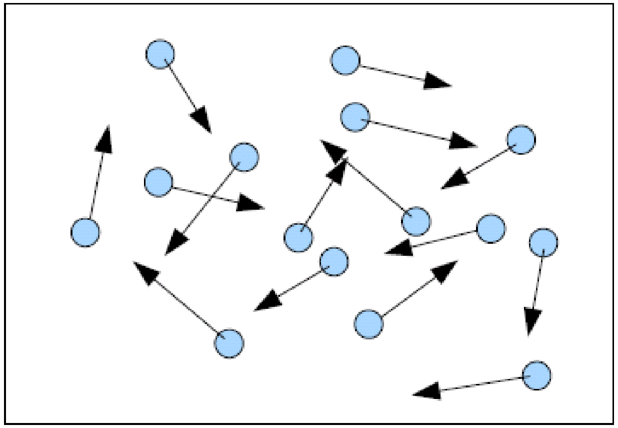
Fairbank et al 1954

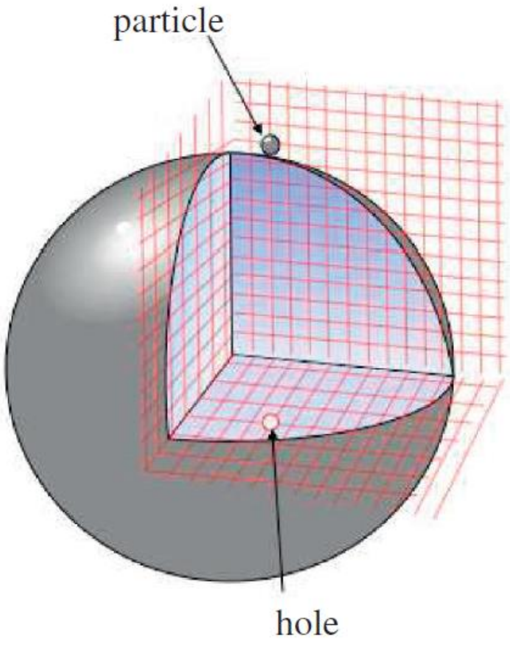
Fairfield et al 1954

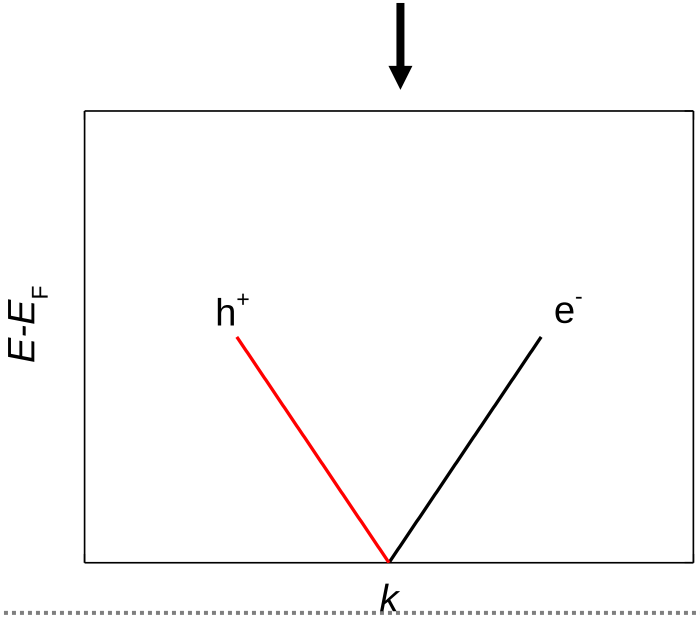
Fermi Liquid Theory

Fermi sea of 3-He atoms

- Ground state
- Consider excitations







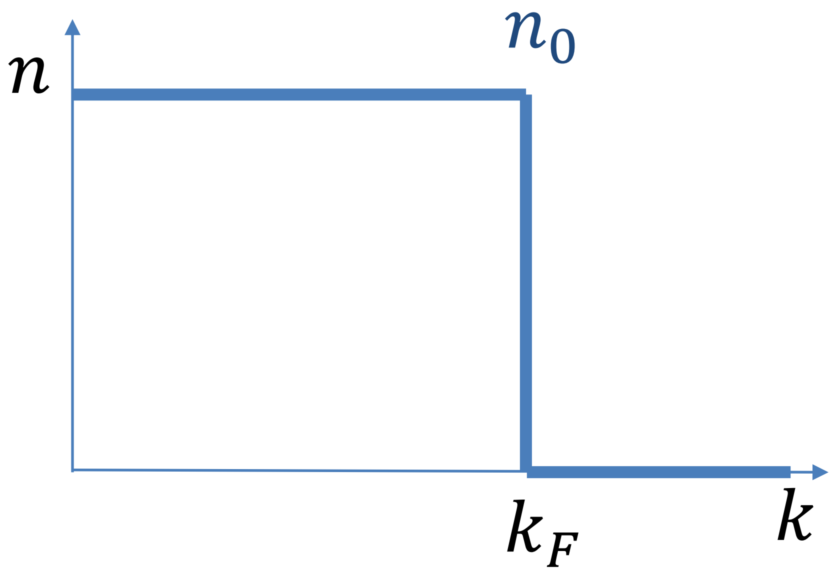
Deviation from ground state $\delta n_k = n_k - n_k^0$

- Non-interacting

$$F - F_0 = \sum_k \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \delta n_k$$

- Interacting

$$F - F_0 = \sum_k \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \delta n_k + O\left(\delta n_k^2\right)$$



Landau Theory

- $F - F_0 = \sum_k (\epsilon - \mu)\delta n_k + \frac{1}{2}\sum'_{kk} f_{kk'}\delta n_k \delta n_{k'}$
- This is an expansion in number of excited states

-> valid for small $\alpha = \frac{\sum_k |\delta n_k|}{N}$

The heart of Landau Theory: interaction terms

$$f_{kk'}$$

Towards Physical Quantities

- Symmetric and antisymmetric contribution

$$f_{kk'}^{\uparrow\uparrow} = f_{kk'}^{s} + f_{kk'}^{a}$$
$$f_{kk'}^{\uparrow\downarrow} = f_{kk'}^{s} - f_{kk'}^{a}$$

- Decomposition into Legendre factors:

$$f_{kk'}^{s(a)} = \sum_{l=1}^{\infty} f_l^{s(a)} P_l(\cos\xi)$$

Landau Parameters

- Dimensionless interaction strength (relative to kinetic energy)

Charge $F_l^s = g(\epsilon_F) f_l^s$

Spin $F_l^a = g(\epsilon_F) f_l^a$

Can be used to calculate physical properties

Physical Properties

- Specfic heat

$$\gamma = \frac{m^* k_F k_B^2}{3\hbar^2}$$

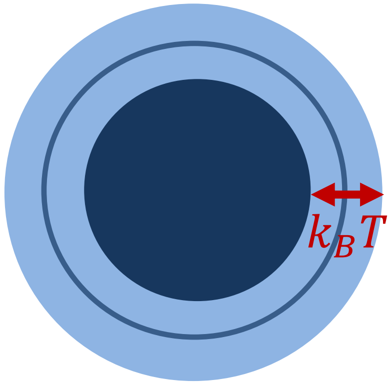
$$\frac{m^*}{m} = 1 + \frac{1}{3} F_1^s$$

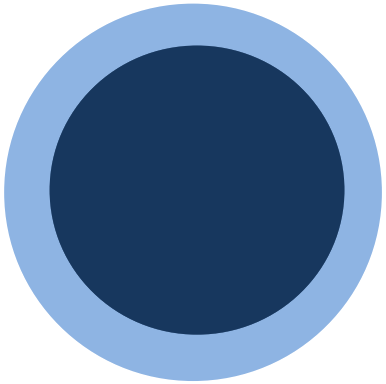
- Compressibility

$$\kappa = \frac{1}{\rho^2} \frac{N(E_F)}{1 + F_0^s}$$

- Sound velocity

$$s^2 = \frac{k_F^2}{3mm^*}(1 + F_0^s)$$

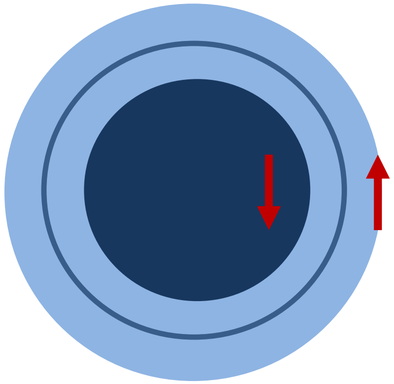




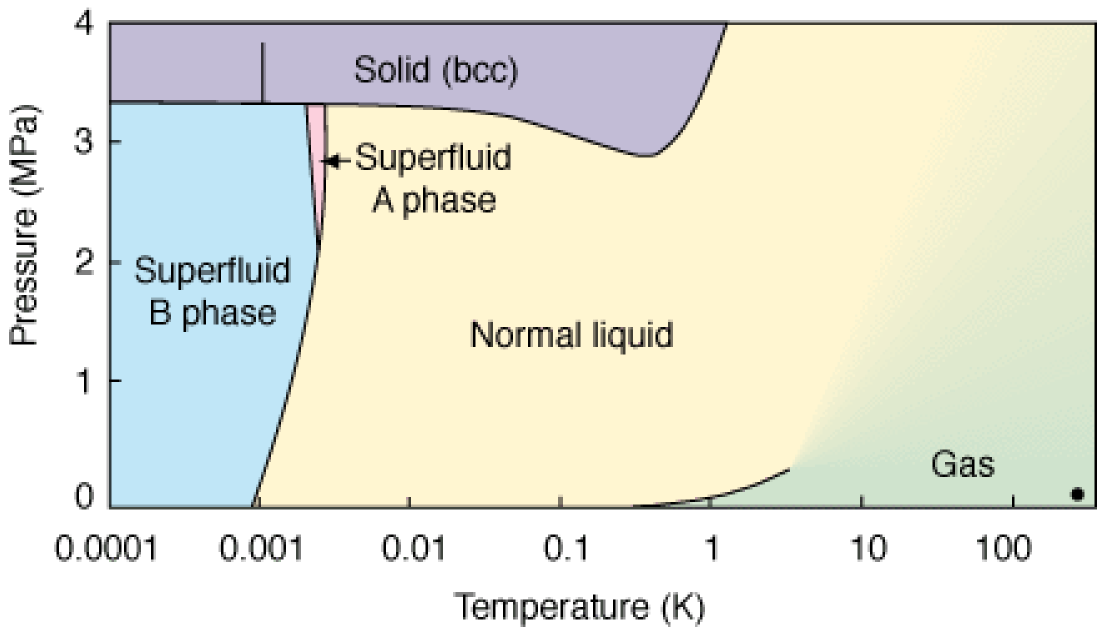
Physical Properties

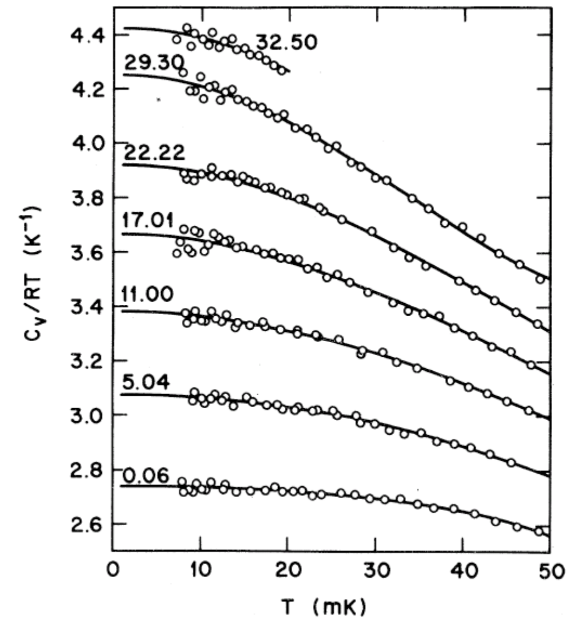
- Spin Susceptibility

$$\chi_P = \frac{\mu_0 \mu_B^2 g(E_F)}{1 + F_0^a}$$



Landau parameters for ^{3}He



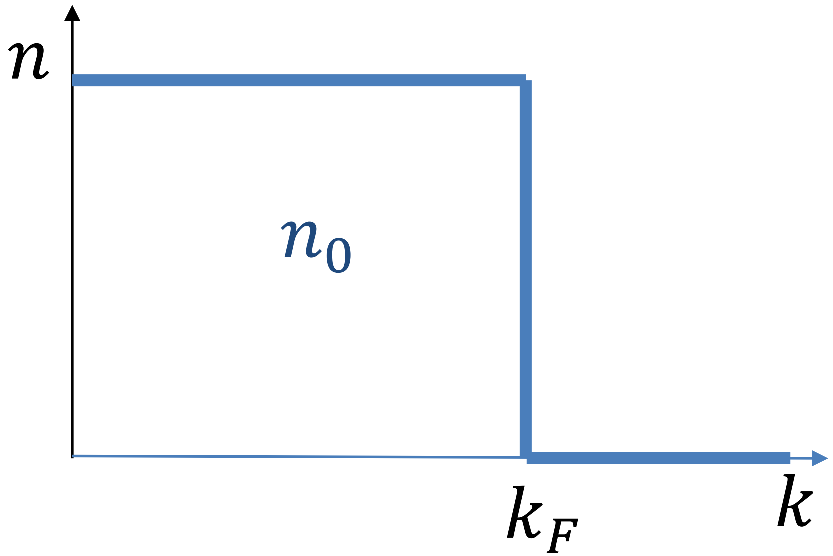


Pressure (atm)	m^*/m	F_0^s	F_0^a	F_1^s
0.28	3.1	10.8	-0.67	6.3
27.0	5.8	75.6	-0.72	14.4

Statistics $n_k^0 = \left(1 + \exp\left[\frac{\tilde{\epsilon}_k - \mu}{k_B T}\right]\right)^{-1}$

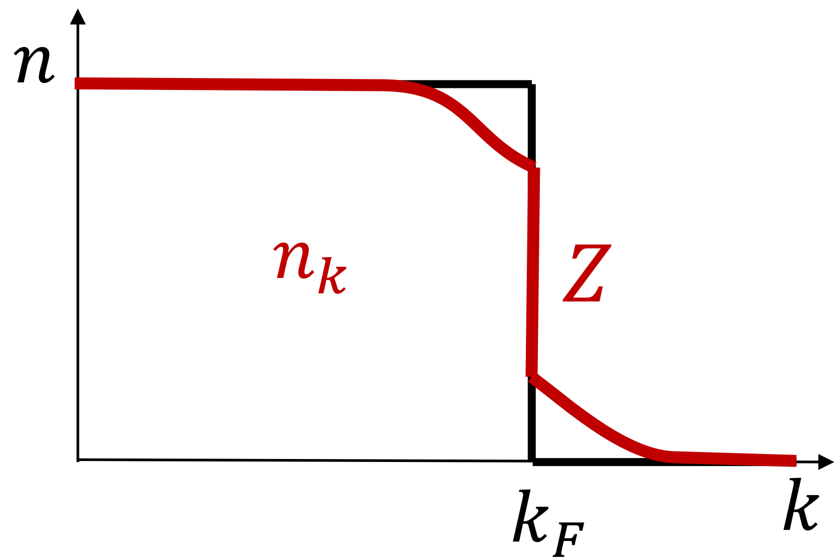
- Non-interacting

$$\tilde{\epsilon}_k = \epsilon_k$$



- Interacting

$$\tilde{\epsilon}_k = \epsilon_k + \sum_{k'} f_{kk'} \delta n_k$$



Z quasiparticle weight

Quasiparticle Concept

A short philosophical excurse

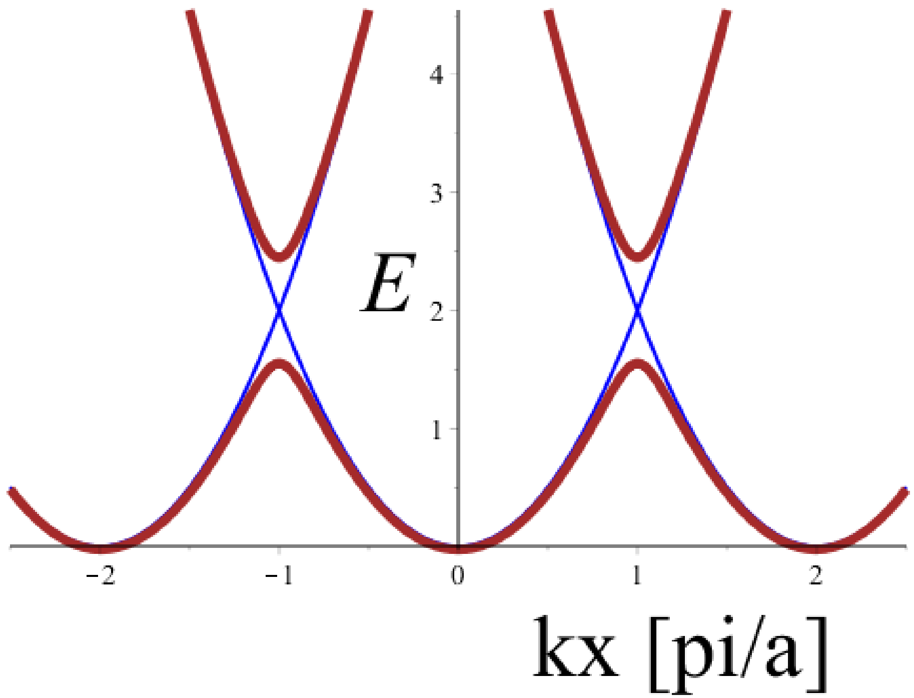
(Incomplete) List of Quasiparticles

Electron	Electronic quasiparticle in metals
Hole	Electronic quasiparticle in metals
Exciton	Bound state of electron and hole
Phonon	Lattice vibration in solids
Roton	Excitation in superfluid Helium 4
Holon	spin separated charge
Spinon	charge separated spin
Soliton	Domain wall
Skyrmion	Topological vortex
Anyon	neither bosons nor fermions
Dirac Fermion	massless fermion

Quasiparticles

! Caution !

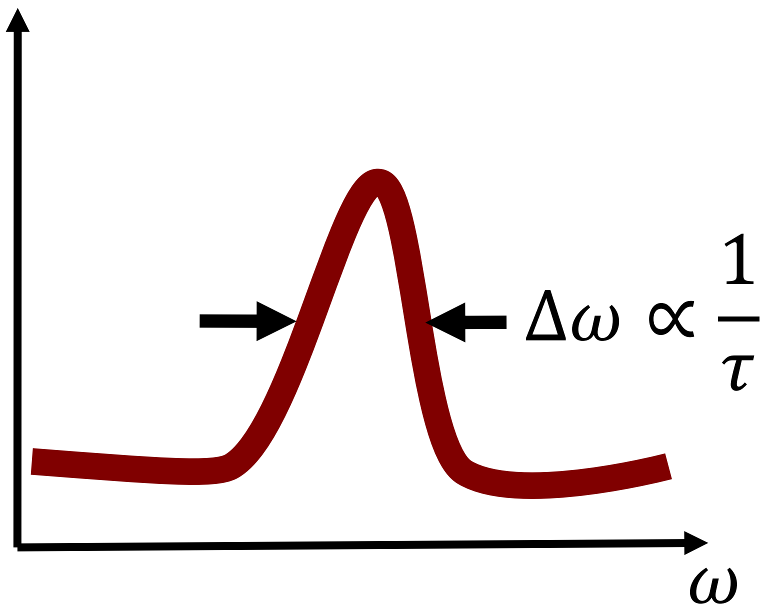
Electron refers to bare electron and electronic quasiparticle



Quasiparticle

Collective Excitation

- Well defined
- Long lived $\frac{1}{\tau} < \omega$



- Model for excitations
- Acts like weakly interacting particle
- Quantum numbers
- Dispersion relation
- Confined to host

$$? \; |0\rangle \; ?$$

Quasiparticle - Relevance

Applications

• Electrons and Holes	metals & semiconductors industry
• Fluxon	SQUID
• Soliton	computing
• Spinon	spintronics
• Skyrmion	data storage

Fundamental Understanding

- Reductionism
- Emergence