Correlated Electron Systems

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Timetable

- Dates and Times:

07/02	2-4.20 pm with 20 min break
21/02	3-4.20 pm with 20 min break
28/02	2-3pm

Overview

0. Free Electron Theory
1. Fermi Liquid Theory
2. Kondo effect and Heavy Fermion Materials
3. Screening, El-Ph Interaction, and RKKY
4. Doniach Picture and Quantum Phase Transitions
5. Non-Fermi-Liquid and Quantum Critical Points
6. (Mott Insulator)

Literature

General Solid state Physics

- C. Kittel, Introduction to Solid State Physics, 7th edition, Wiley, NY, 1996.
- N.W. Ashcroft and N.D.Mermin, Solid State Physics, 1976.
- J.M. Ziman, Principles of the Theory of Solids, CUP, Cambridge, 1972.
- H. Ibach and H. Lüth, Solid State Physics, Springer 1995.
- J. Singleton, Band Theory and the Electronic Properties of Solids, OUP 2001.

Correlated electron materials

- Coleman, P. *Introduction to Many-Body Physics*. CUP 2015
- Y. Onuki *Physics of Heavy Fermions*
- S. Sachdev, *Quantum Phase Transitions.* CUP 1998
- A. C. Hewson, *The Kondo problem to heavy fermions*. CUP, 1997.
- Pines and Nozieres: The Theory of Quantum Liquids

Correlated Electron Systems

0) (Nearly) Free electron Theory

Drude model - Successes

- Gas of independent electrons
- Collisions with ionic cores only
- Relaxation time approximation

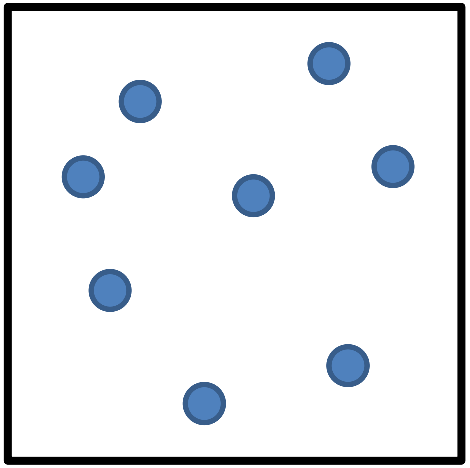
Electrical Transport

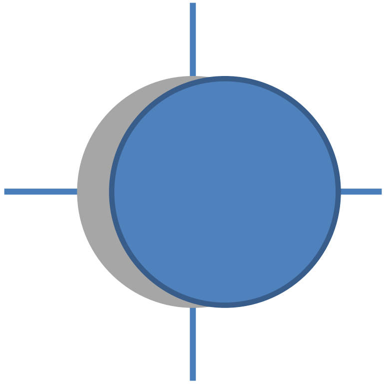
$$j = -nev_D = -\frac{ne}{m}p_D$$

$$0 = \frac{dp_D}{dt} = -eE - \frac{p_D}{\tau}$$

$$= -eE - \frac{mv_D}{\tau}$$

$$\sigma = \frac{j}{E} = \frac{ne^2\tau}{m}$$





Reason:
Full Fermi
surface displaced

Conductivities – Wiedemann Franz

Electrical conductivity

$$\sigma = \frac{ne^2\tau}{m^*}$$

Thermal conductivity

$$\kappa = \frac{1}{3}C\langle c\rangle l$$

$$= \frac{\pi^2 n k_B^2 T\tau}{3m^*}$$

Wiedemann-Franz Law

$$\frac{\kappa}{\sigma} = \frac{\pi^2 k_B T\, n\, \tau/3m^*}{ne^2\tau/m^*} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 T = L_0 T$$

Lorentz number $L = 2.45 * 10^{-8}\frac{\mathrm{W\Omega}}{\mathrm{K}}$

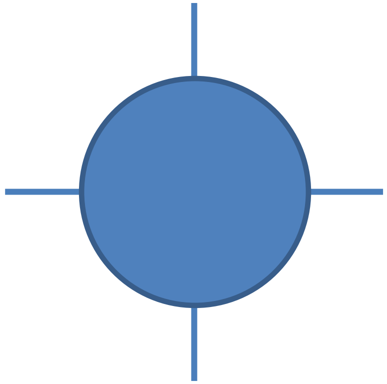
Drude Model - failures

Specific heat: Orders of magnitude to large

$$C = \frac{3}{2} n k_B$$

Reason: Not all electrons can be excited

Solution: Sommerfeld model



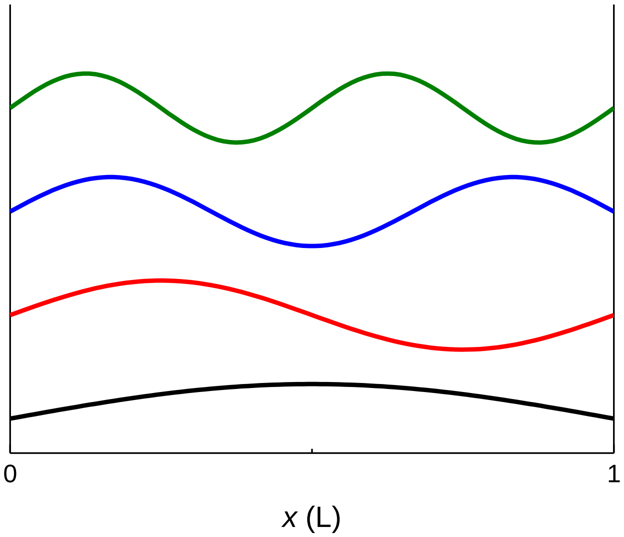
Sommerfeld Model

- Quantized states
- k – space volume per state

$$\frac{(2\pi)^3}{V}$$

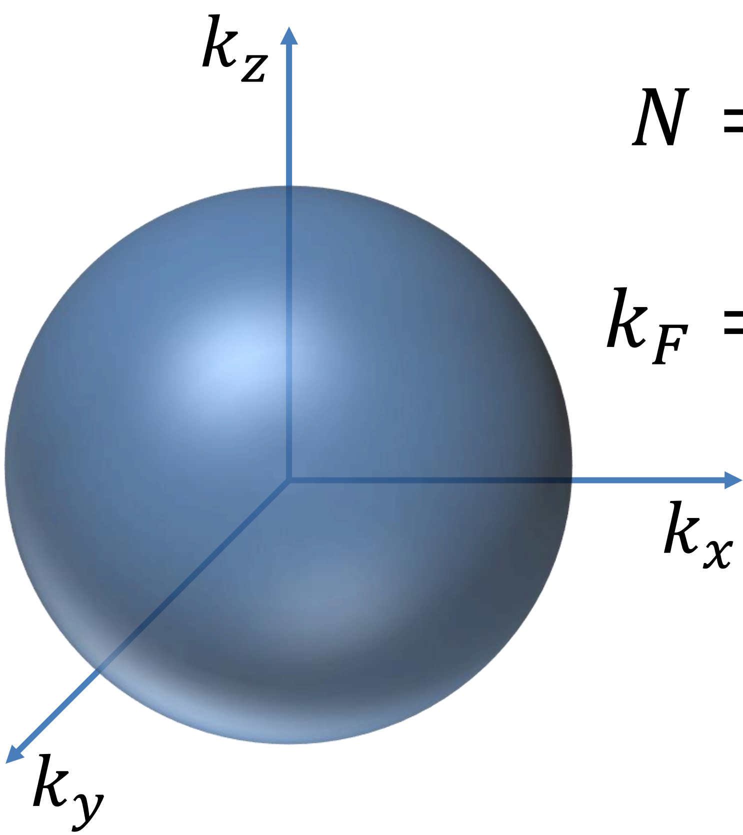
- Electron energy $\mathrm{E}(k) = \frac{\hbar^2 k^2}{2m_e}$
- Fermi-Dirac Statistics

$$f(E) = \frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right)+1}$$



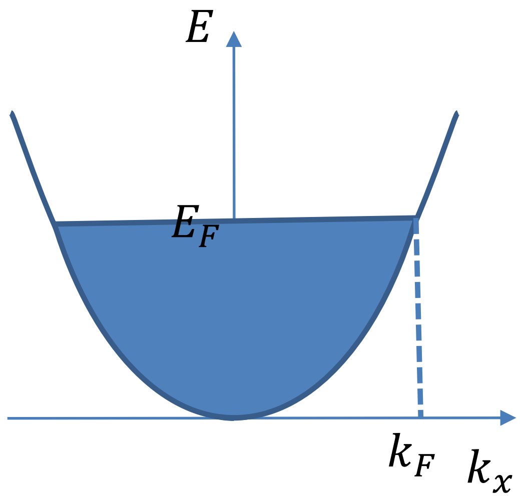
Free Electron Model

- Electrons fill low energy states up to E_F



$$N = 2\frac{4/3\pi k_F^3}{(2\pi/L)^3}$$

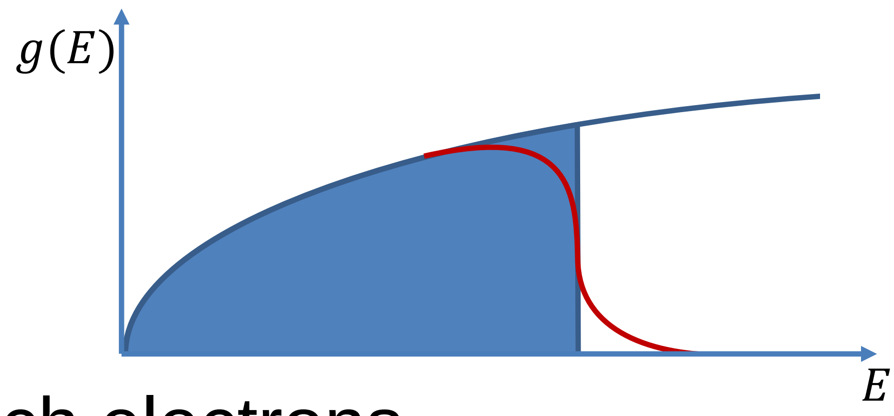
$$k_F = (3\pi^2 n)^{1/3}$$



3D $g(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E}$

Specific Heat (qualitatively)

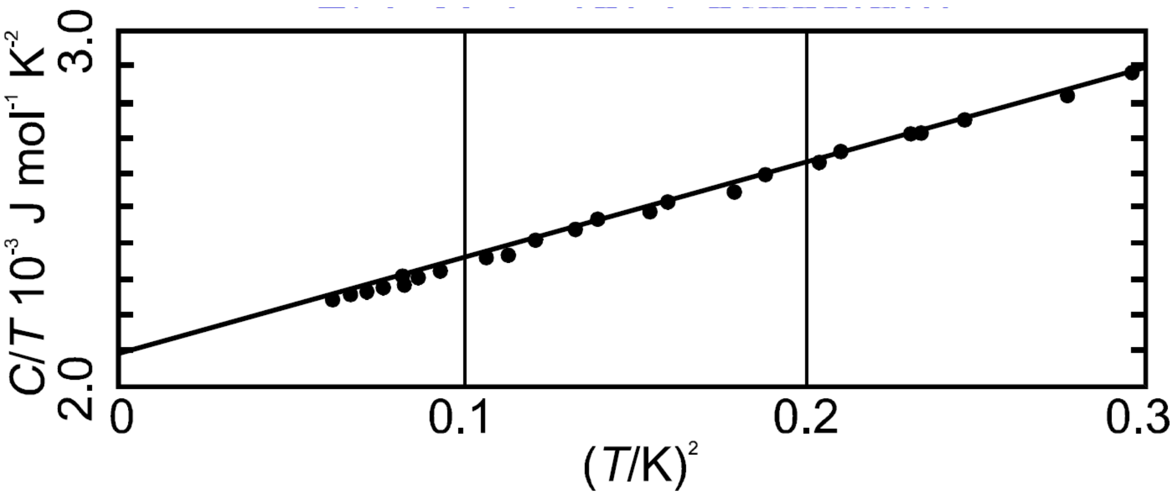
$$C_{el} = \frac{dU_{el}}{dT}$$



- Electrons within $k_{\mathrm{B}}T$
- There are $g(E_{\mathrm{F}})k_{\mathrm{B}}T$ such electrons
- With (classical kinetic) energy $E = \frac{3}{2}k_B T$
- $U_{el} \propto g(E_{\mathrm{F}})T^2$
- $C_{el} \propto g(E_{\mathrm{F}})T$ exact: $C_{el} = \gamma T$
- Sommerfeld coeffecient $\gamma = \frac{\pi^2}{2} n k_B \frac{1}{T_F}$

Sommerfeld coefficient

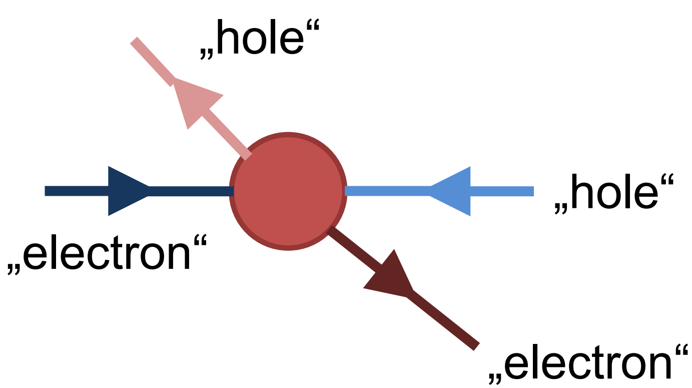
- $C = \gamma T + \beta T^3 \rightarrow \frac{C}{T} = \gamma + \beta T^2$
  (βT^3 - Lattice specific heat)

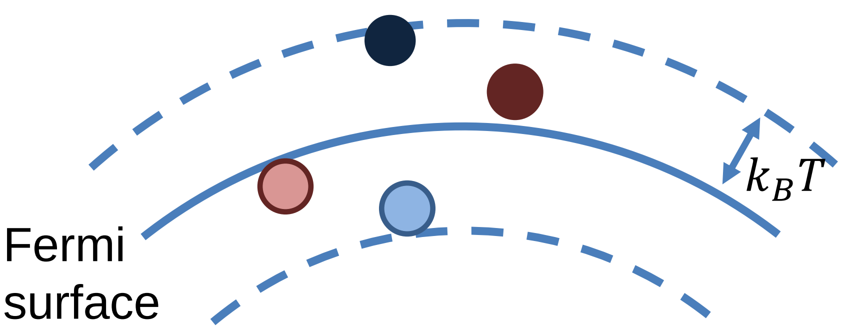


Potassium (K)	Theoretical	Experimental
γ(mJ mol K^{-2})	1.67	2.08

Electron scattering rate

- Scattering event:
- Energy, momentum, and charge conservation
- Number of initial states $\propto [g(E_F) * (E - E_F)]$
- Number of final states $\propto [g(E_F) * (E - E_F)]$





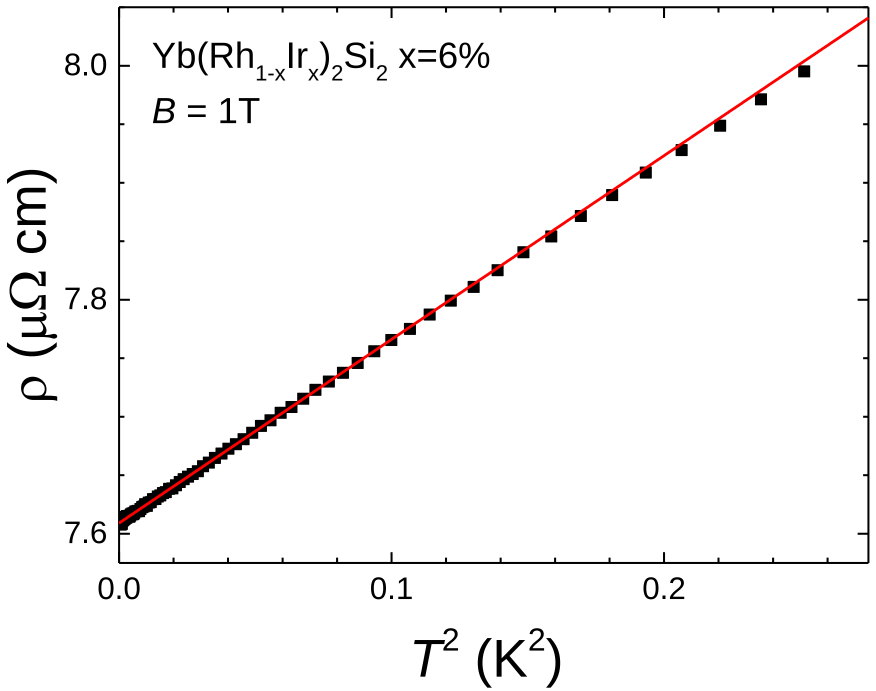
- Scattering rate $\Gamma \propto [g(E_F) * (E - E_F)]^2$

Electrical Resistivity

- Lifetime $\tau \propto [g(E_F) * T]^{-2}$
- Boltzman theory $\rho \propto \frac{1}{\tau} \propto g(E_F)^2 T^2$
- With residual resistivity

$$\rho = \rho_0 + AT^2$$

- Prefactor $A \propto g(E_F)^2$ small for normal metals
- Large detected in heavy fermion materials with large DoS



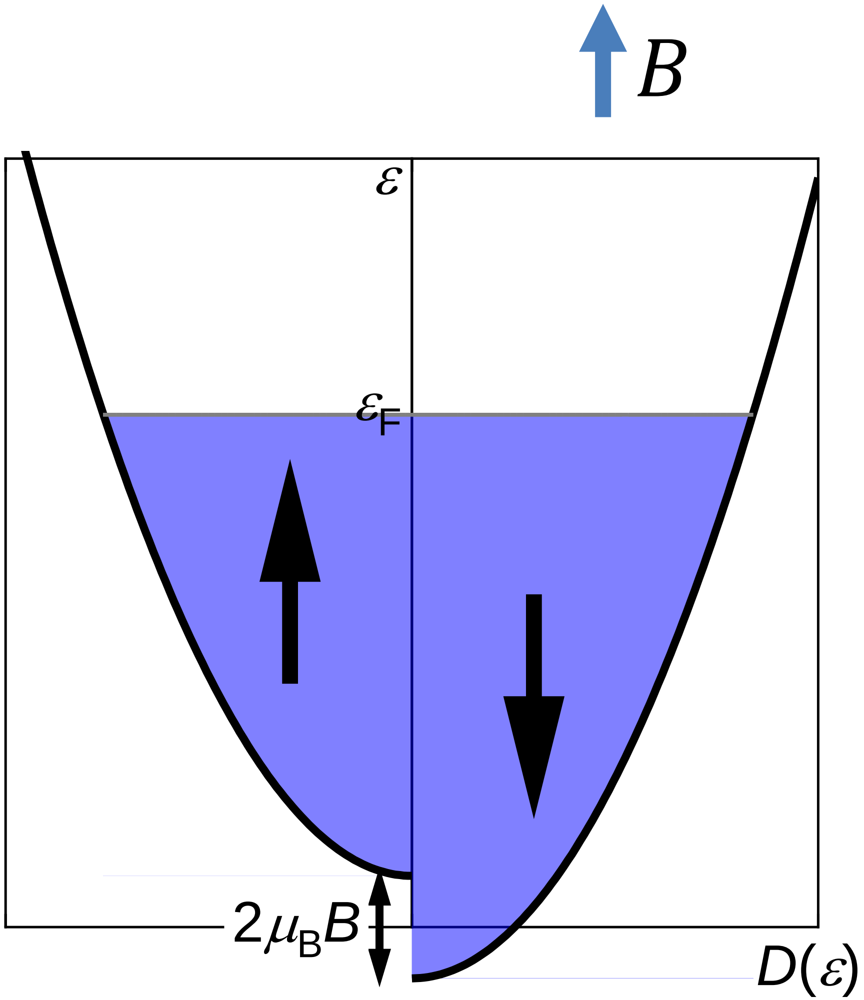
Susceptibilty

$$N^{\uparrow}_{\downarrow}(\epsilon) = \frac{1}{2} N(e \mp \mu_0 \mu_B H)$$
$$= \frac{1}{2}(N \pm g(E_F)\mu_0\mu_B H)$$

$$M = \left(N^{\uparrow} - N^{\downarrow}\right)\mu_B$$
$$= g(E_F)\mu_0\mu_B^2 H$$

- Pauli susceptibility

$$\chi_P = \frac{\partial M}{\partial H} = \mu_0 \mu_B^2 g(E_F)$$



Kadowaki Woods ratio

- Specific heat

$$\frac{C}{T} = \gamma + \beta T^2$$

$$\gamma = \frac{1}{3}\pi^2 k_B^2 \boldsymbol{g}(\boldsymbol{E_F})$$

- Resistivity

$$\rho = \rho_0 + AT^2$$

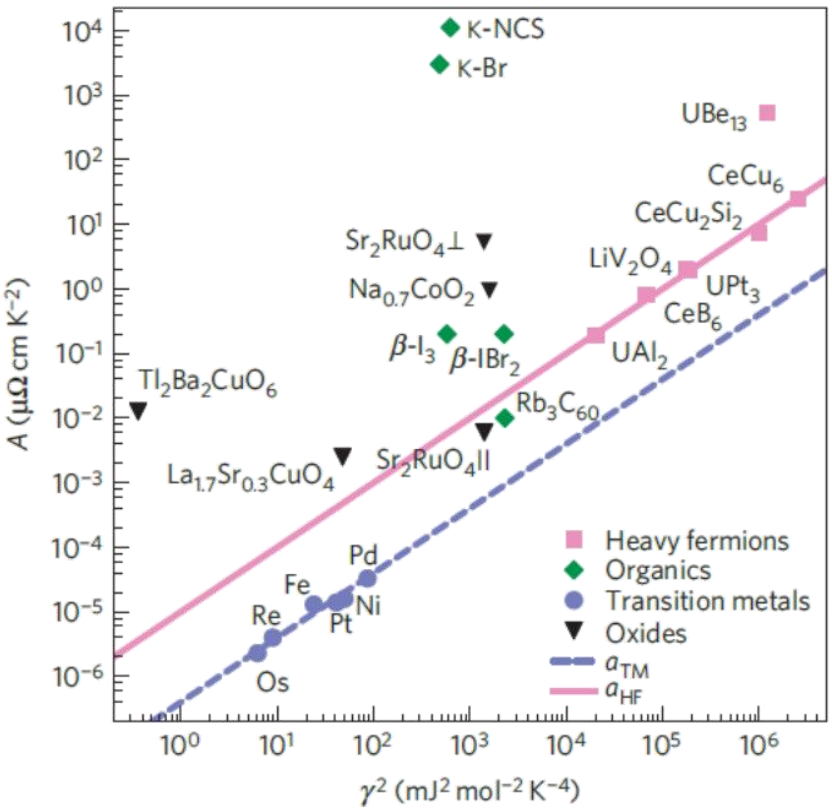
$$A \propto \boldsymbol{g}(\boldsymbol{E_F})^{\mathbf{2}}$$

Kadowaki, K. & Woods, S. B. *Solid State Commun.* **58,** 507 (1986)

Nature Physics volume 5, 422 (2009)

- Kadowaki-Woods ratio

$$\frac{A}{\gamma^2} = const \approx 10\ \mu\Omega\ \mathrm{cm\ mol^2 K^2 J^{-2}}$$



Sommerfeld-Wilson Ratio

- Specific heat

$$\frac{C}{T} = \gamma + \beta T^2$$

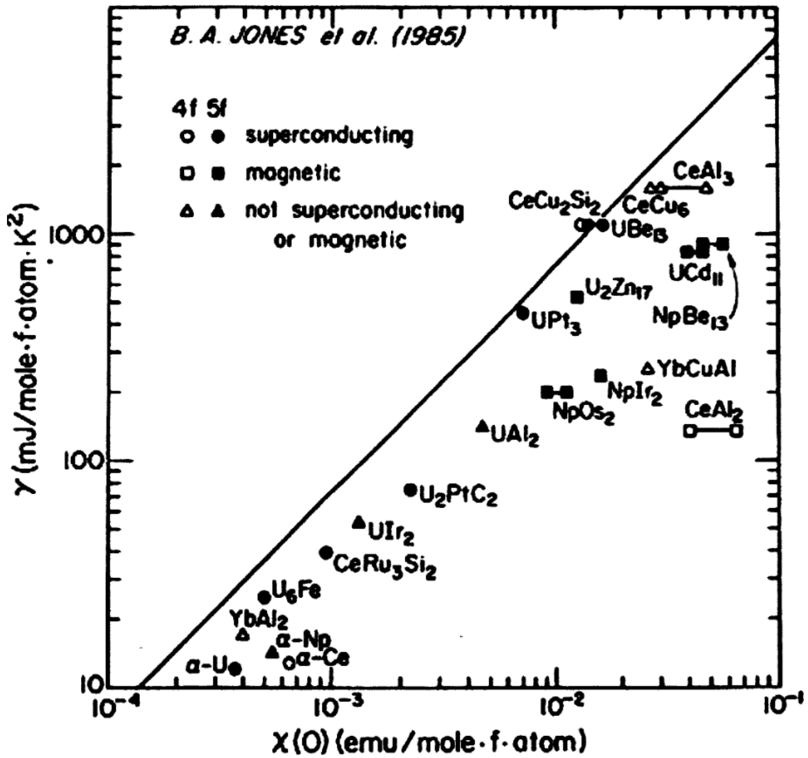
$$\gamma = \frac{1}{3}\pi^2 k_B^2 \boldsymbol{g}(\boldsymbol{E_F})$$

susceptibility

$$\chi_P = \mu_0 \mu_B^2 \boldsymbol{g}(\boldsymbol{E_F})$$

- Sommerfeld-Wilson Ratio

$$SWR = \frac{\chi_0}{\gamma_0} = \frac{3\mu_0\mu_{eff}^2}{\pi^2 k_B^2}$$



Lee, P. A., et al. *Comments Condens. Matt. Phys.* **12,** 99 (1986).

Sommerfeld - limitations

- Hall effect

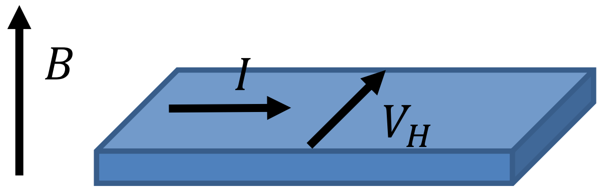
$$R_H = -\frac{1}{ne}$$

Opposite sign observed

Reason: negative band curvature can cause apparent -tive mass

$$\frac{\partial^2 \epsilon}{\partial k^2} < 0$$

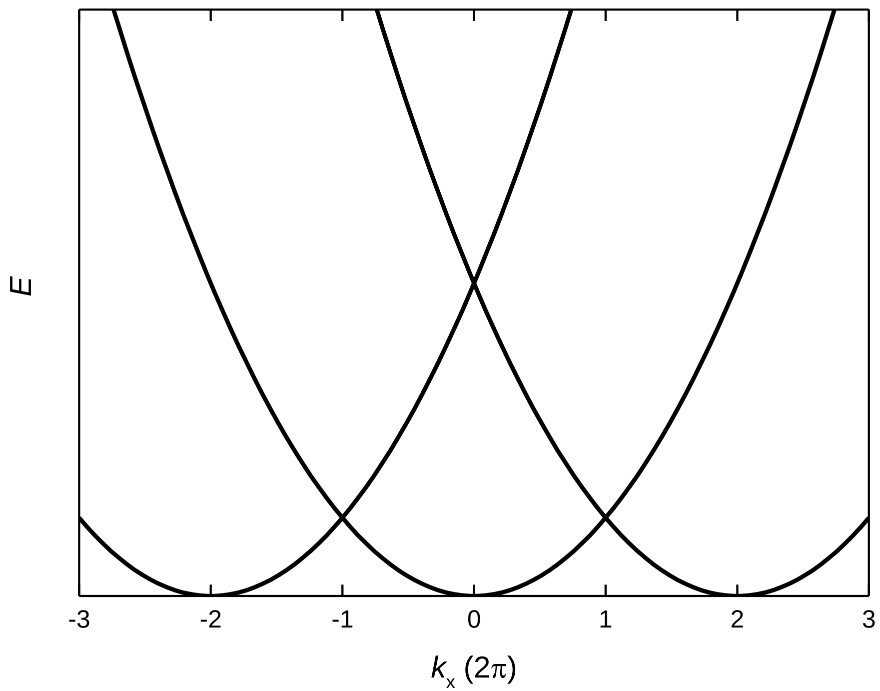
Solution: Periodic potential

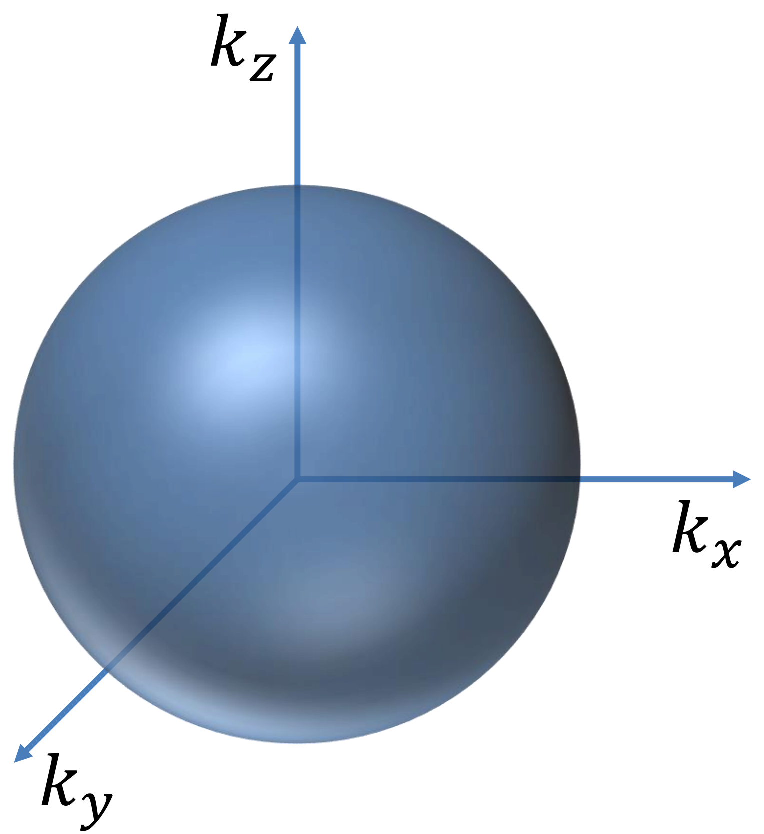


Fermi Surface

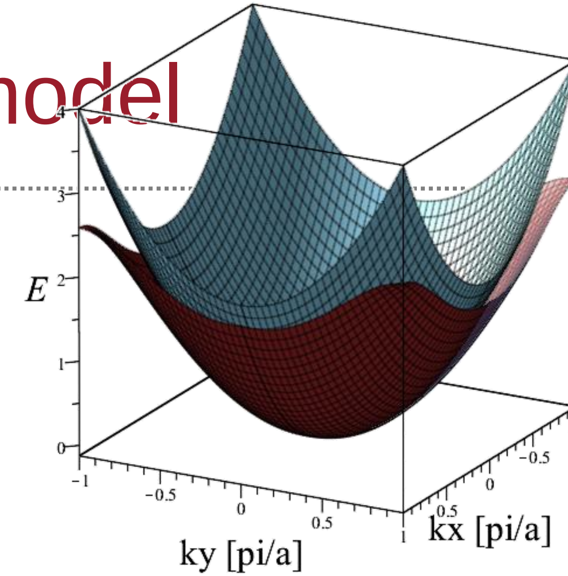
- Free electron

$$E(\boldsymbol{k}) = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right)$$

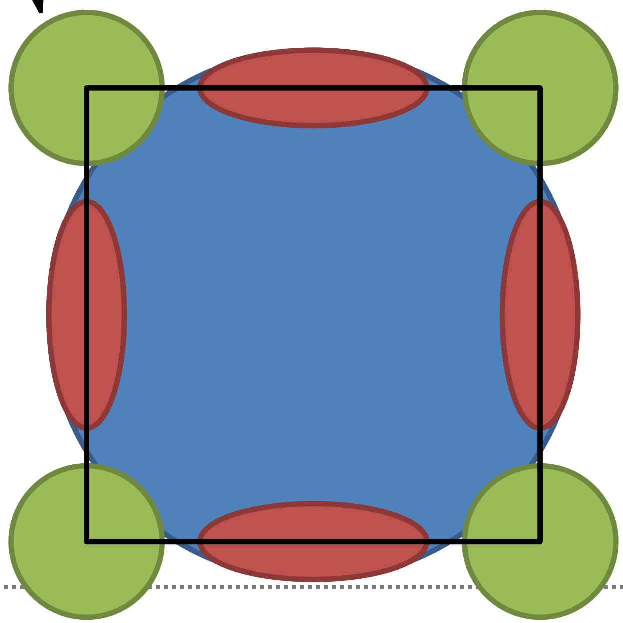


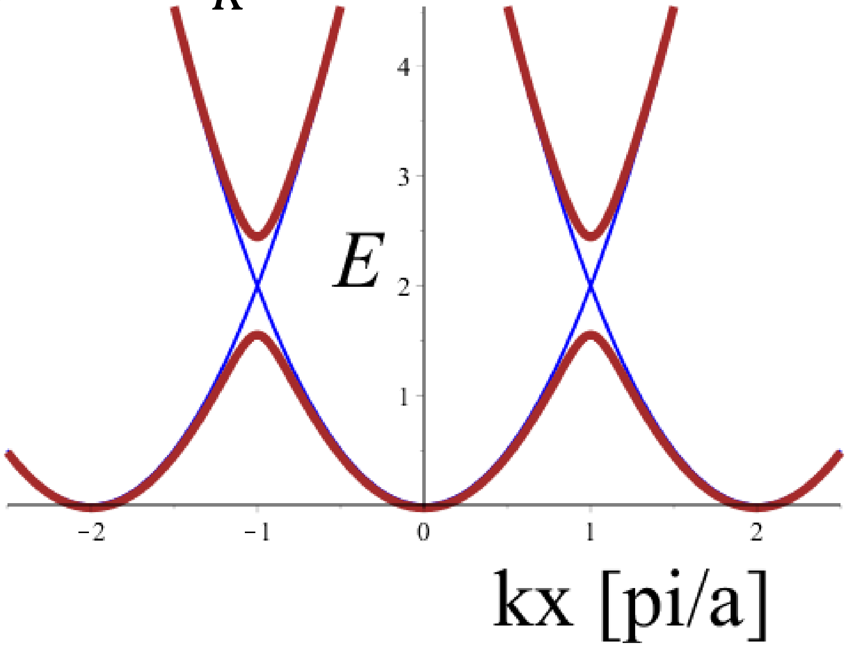


Nearly free electron model

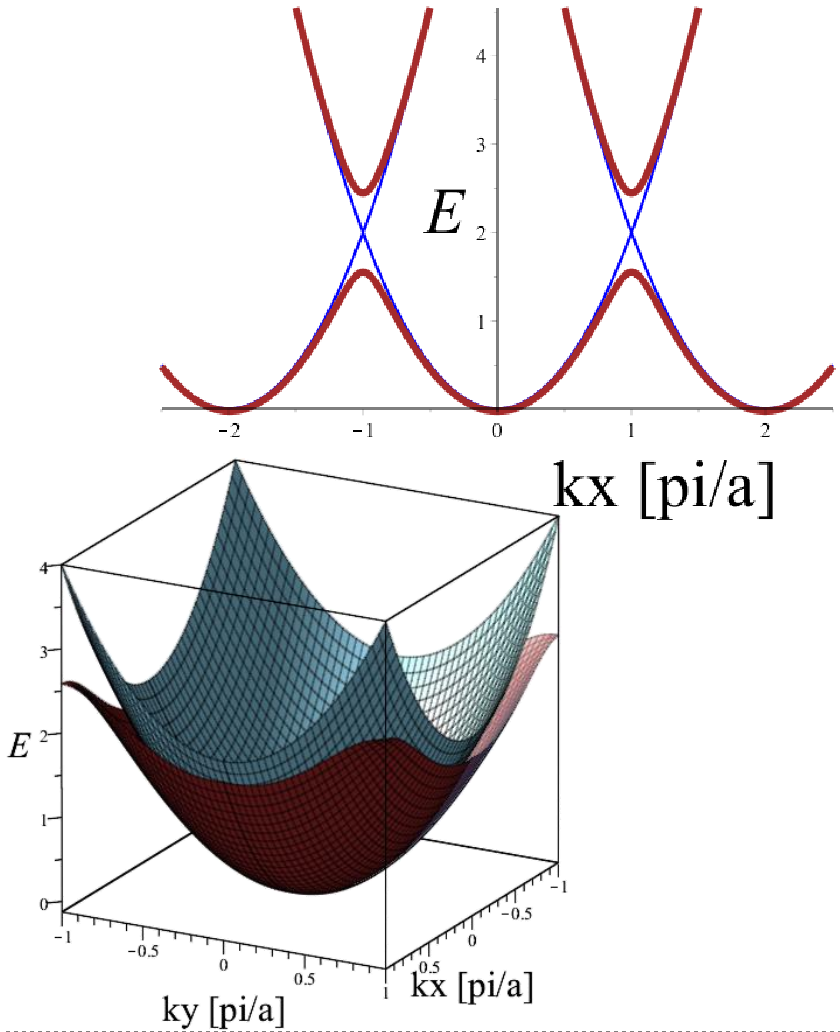


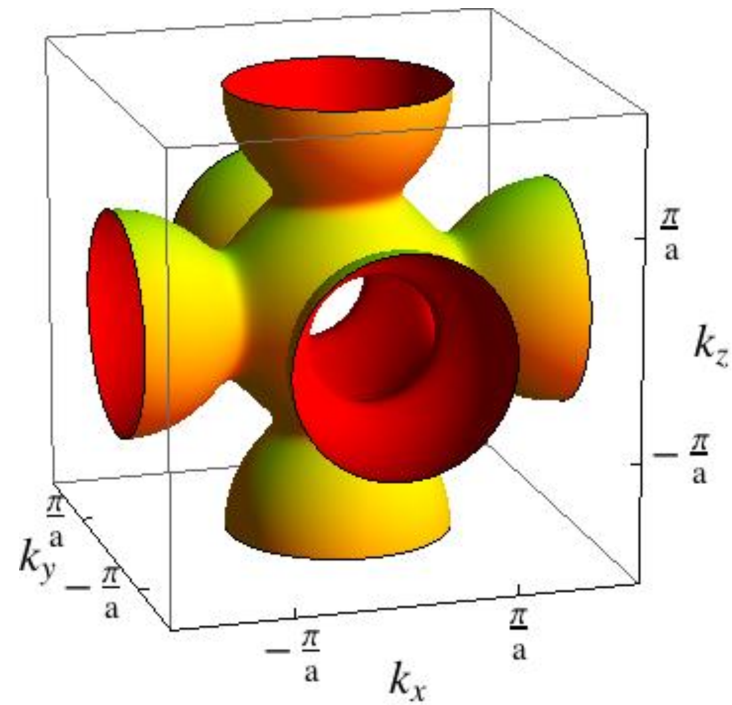
$$E(\boldsymbol{k}) = \frac{\hbar^2}{2m}\frac{1}{2}\left(k^2 + (k-K)\right)^2 \pm \frac{1}{2}\sqrt{\left(\frac{\hbar^2}{2m}k^2 - \frac{\hbar^2}{2m}(k-K)^2\right)^2 + 4V_K^2}$$





Nearly Free electron model

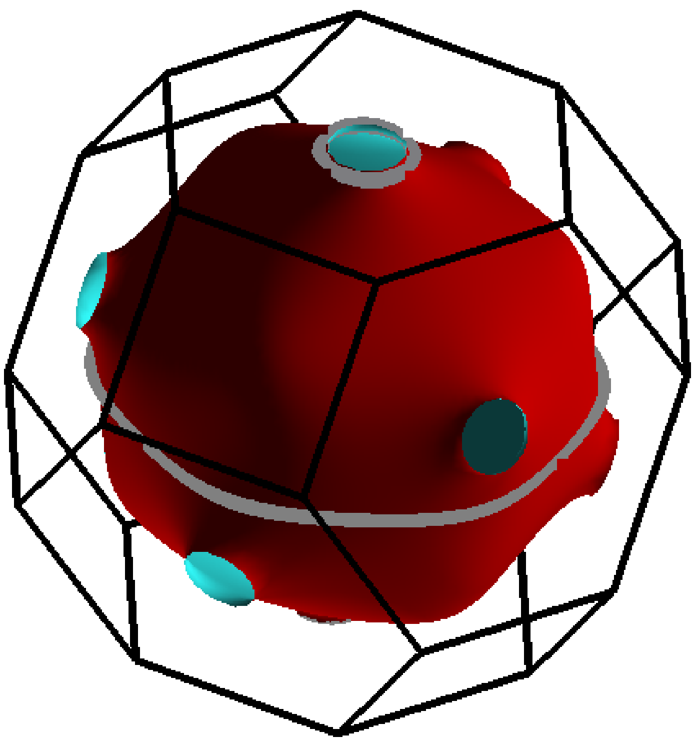




Fermi surface measurement

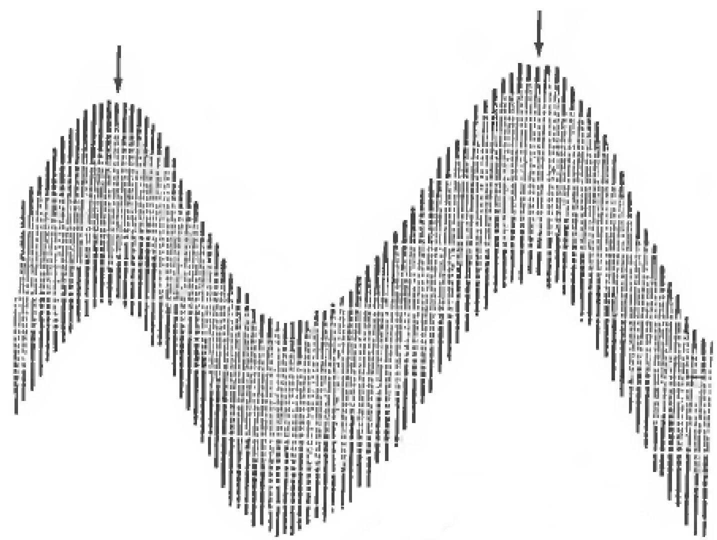
Frequency of extremal orbit

$$F = \frac{\hbar}{2\pi e} A$$



Quantum Oscillation measurements

- Prove Fermi Surface
- Probe extremal orbits
- Measure effective mass



Successes of (nearly) Free Electrons

- Electrical transport
- Heat capacity
- Thermal transport
- Ultrasound
- Spectroscopy
- Band structure
- Fermi Surface
- …

Basis of electronic engineering

- Transistor
- Integrated electronics
- Computer
- Sensors
- LEDs
- …

Limitations of Nearly Free Electron Model

- No many-body states
- Single-particle Hamiltonian (with periodic potential)
- Filled with independent states (Fermi-Dirac Statistics)
- Coloumb interaction neglected

-> shouldn‘t work at all

How comes that we get so good agreement with experiment?