An optimal framework for assessing uncertain parameters in large-scale composites using nonlinear MS-GFEM.

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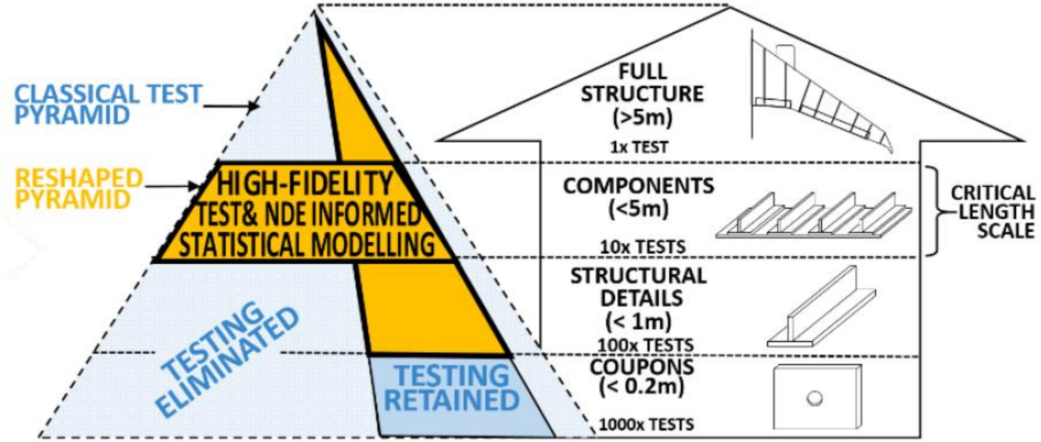


Problem:

- Multi-scale problem
    - Resolve stress distribution at the imperfection level
    - Structural effect (Nonlinear Geometry)
    - Nonlinear effect (Material NL, CZ)

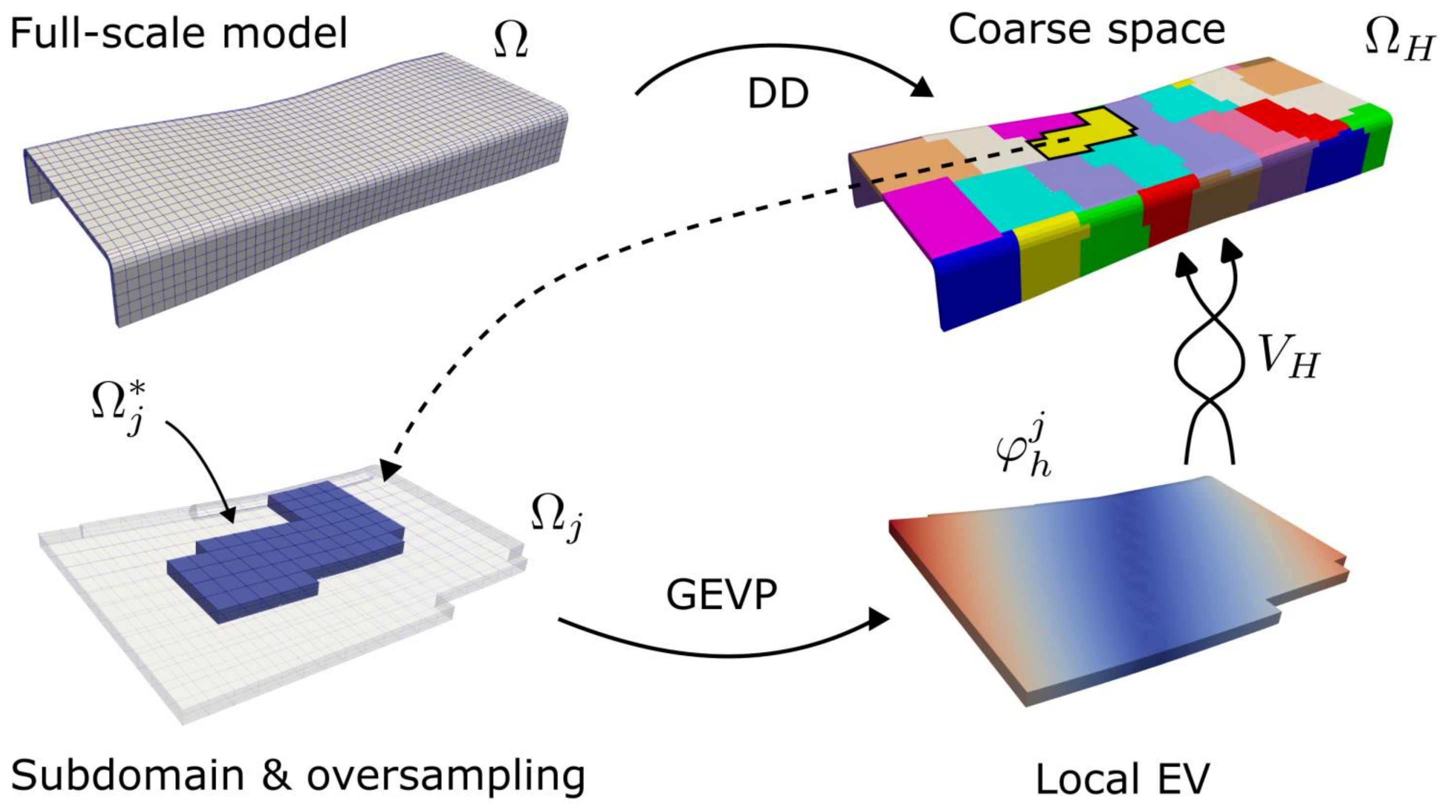
- Uncertainty quantification (UQ) problem
    - Sub-component and upper levels
    - Virtual testing

Method objectives:

- Multi-scale method designed for UQ
- Imperfection assessment – No scale separation
- Leverage parallelisation
- Adapted to aerospace composites
- Not restricted by commercial software
- Implicit method

http://dune-project.org/

Multi-scale Spectral Generalized Finite Element Method (**MS-GFEM**):

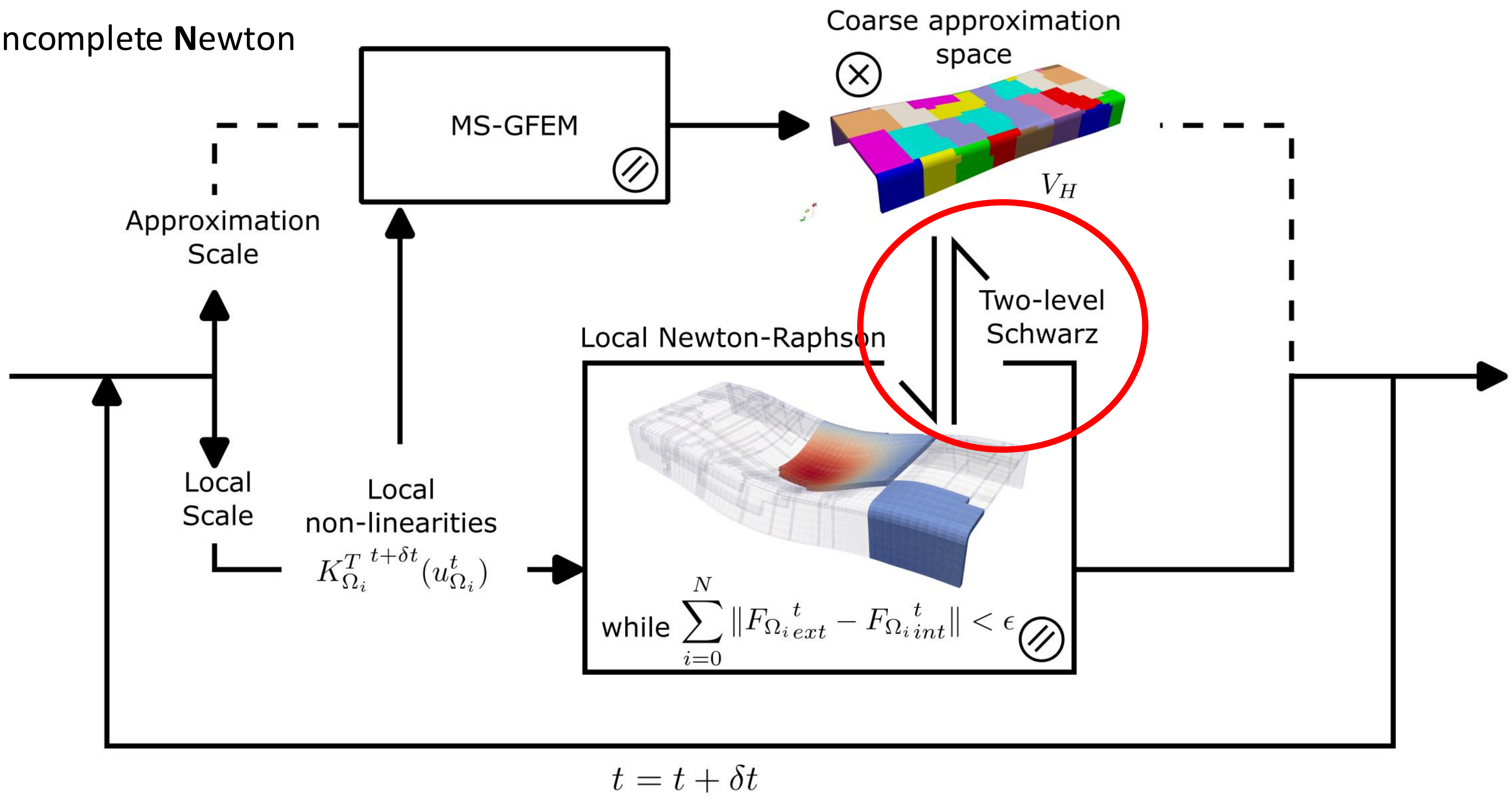


- **Spectral Reduced Order Model**
- **Partition of Unity** operator to stitch together subdomains
- **Oversampling** improves accuracy of coarse solution at interfaces
- **A-harmonic** optimal subspace
  - $\varphi_h^{j,k}$ mechanically admissible
- **Scalable**

Bénézech, J, et al. Scalable multiscale-spectral GFEM with an application to composite aero-structures, 2023, accepted manuscript to JCP https://doi.org/10.48550/arXiv.2211.13893

Optimal nonlinear framework: RASPIN type with spectral approximation space

Inexact and Incomplete Newton

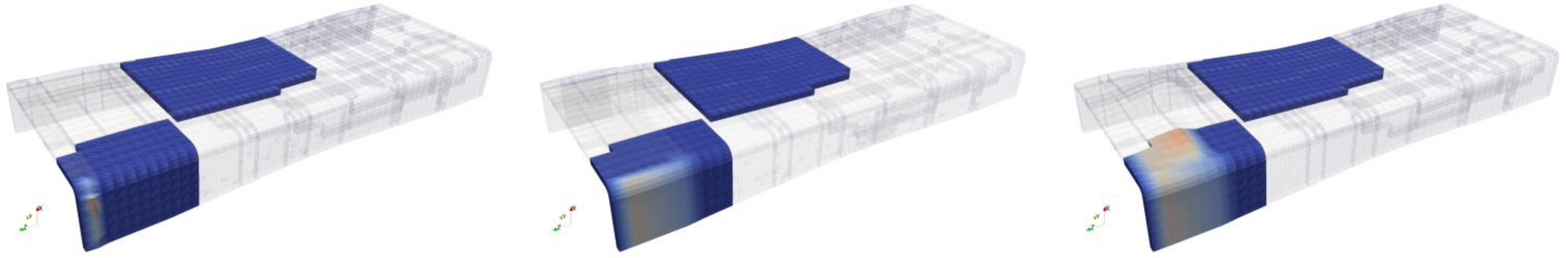


Optimal nonlinear framework: RASPIN type with spectral approximation space

Restricted **A**dditive Two-Level **S**chwarz

Local space

Residual | Particular Solution | Parallel communication



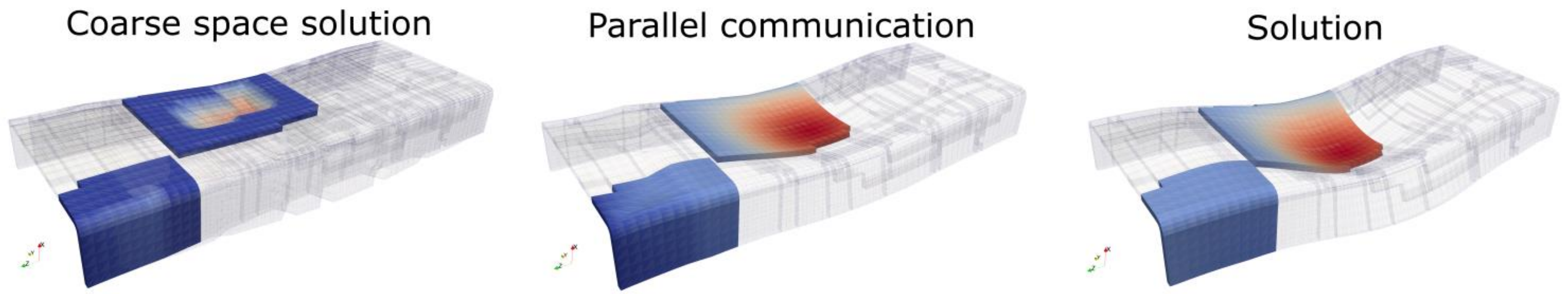
$$\|R_{\Omega_i}\| = \|F_{\Omega_i\, ext} - F_{\Omega_i\, int}\| \qquad K^T_{\Omega_i}\,\Psi_{\Omega_i} = R_{\Omega_i} \qquad \Xi_i(\Psi_{\Omega_i}) \qquad R_{\Omega_i\, corr.} = R_{\Omega_i} - K^T_{\Omega_i\,\mathtt{Neumann}}\Psi_{\Omega_i}$$

Dolean, V., Nataf, F., Scheichl, R., & Spillane, N. (2013). A two-level Schwarz preconditioner for heterogeneous problems. In *Domain Decomposition Methods in Science and Engineering XX* (pp. 87-94). Springer Berlin Heidelberg.

Optimal nonlinear framework: RASPIN type with spectral approximation space

Restricted **A**dditive Two-Level **S**chwarz

Approximation space



$$R_H = f|_{V_H}(R_{\Omega_i\, corr.})$$

$$K_H^T\ \Delta U_H = R_H$$

$$\Delta U_{\mathtt{corr.}\,i} = f|_{V_H}{}^{-1}(\Delta U_H)$$

$$\Xi_i(\Delta U_{\mathtt{corr.}\,i})$$

$$\Delta u_{\Omega_i} = \Psi_{\Omega_i} + \Delta U_{\mathtt{corr.}\,i}$$

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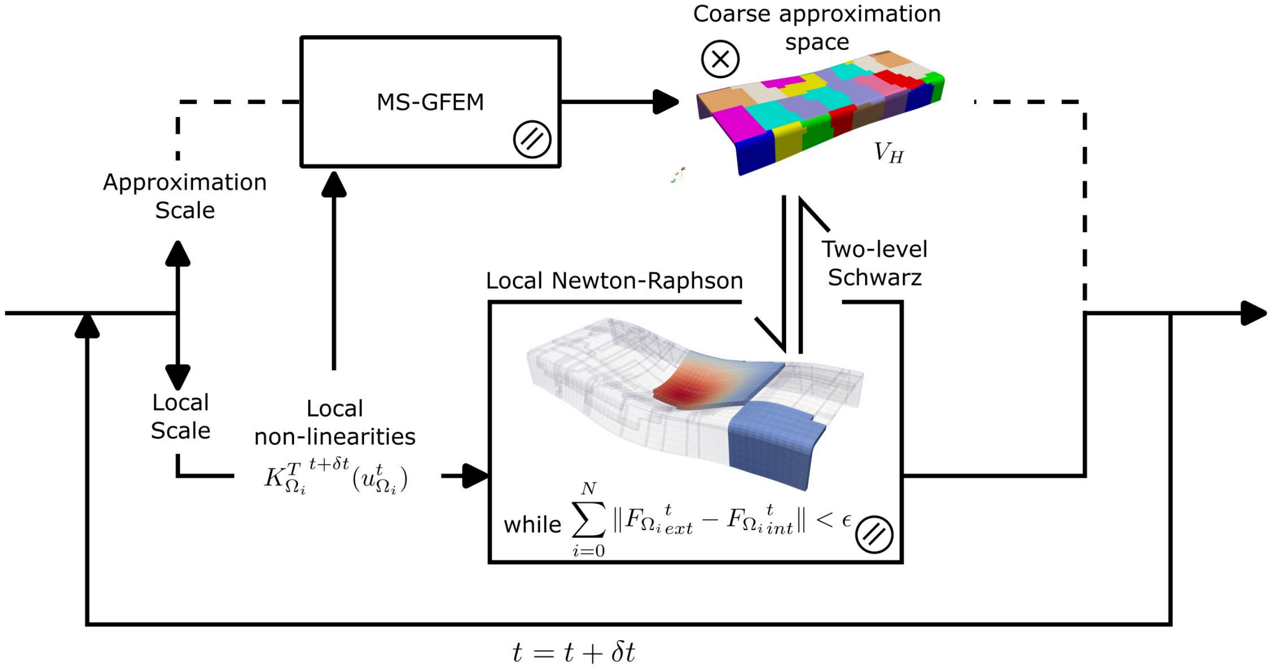


Strongly parallelisable

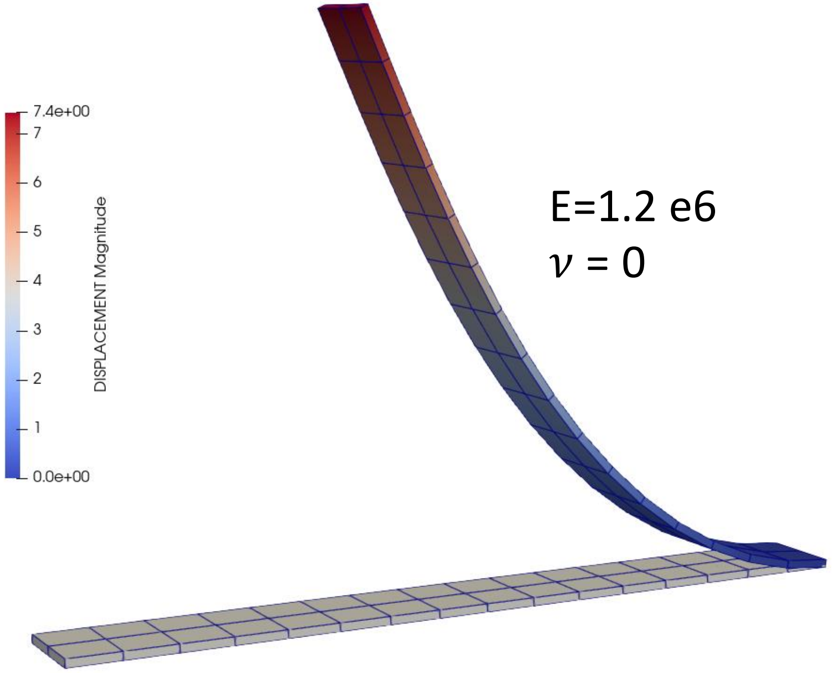
- Local GEVP //
- Local particular solution //
- Coarse space construction needs communication
- Gathering subdomain contributions needs communication

Geometric non-linearity

REF: Sze, K. Y., Liu, X. H., & Lo, S. H. (2004). Popular benchmark problems for geometric nonlinear analysis of shells. *Finite elements in analysis and design*, *40*(11), 1551-1569.

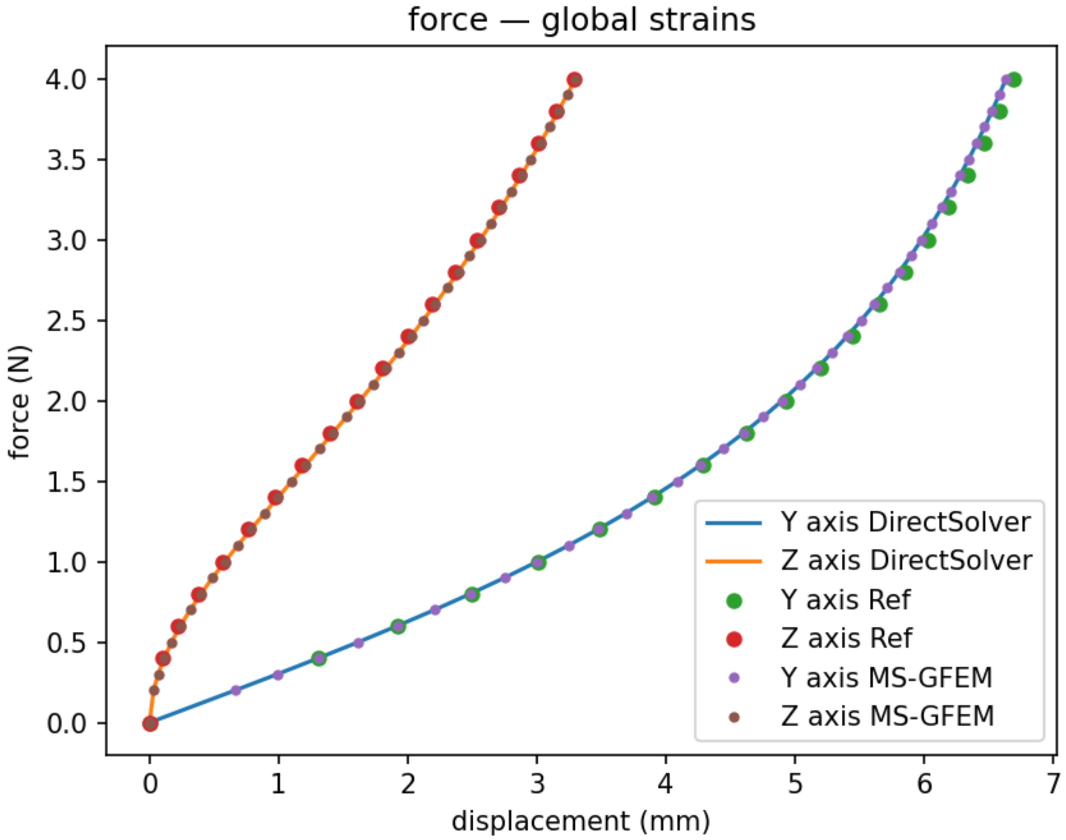
Cantilever beam bending test

4 N



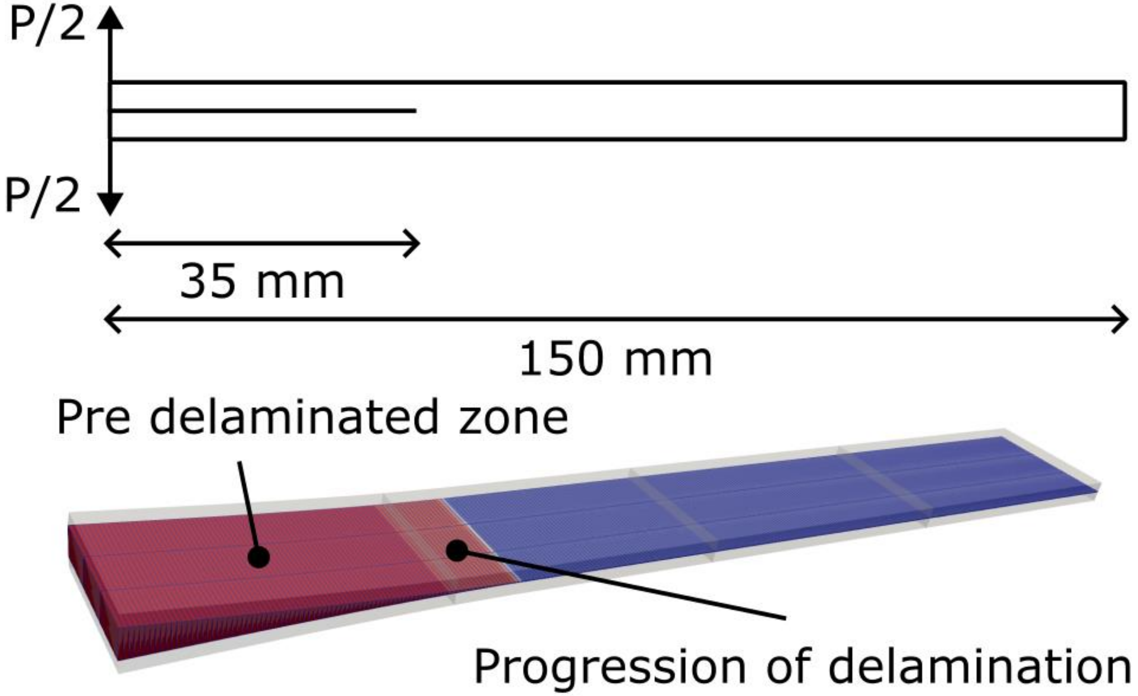
E=1.2 e6
$\nu = 0$

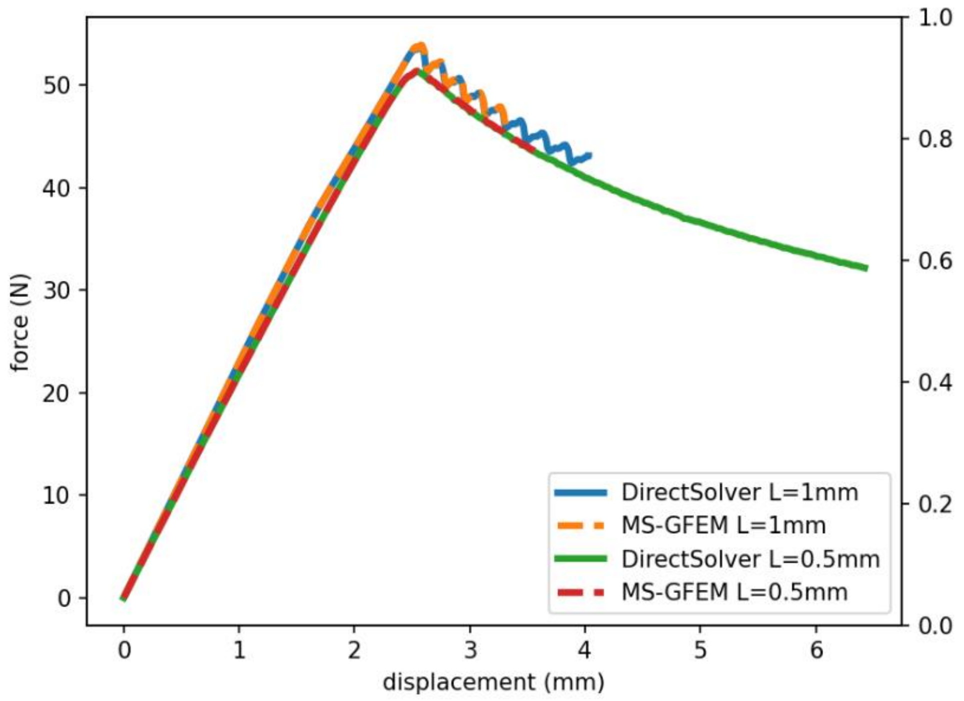
Quadratic serendipity elements



Cohesive element implementation

Harper, P. W., & Hallett, S. R. (2008). Cohesive zone length in numerical simulations of composite delamination. *Engineering Fracture Mechanics*, *75*(16), 4774-4792.





Turon, A., Camanho, P. P., Costa, J., & Dávila, C. G. (2006). A damage model for the simulation of delamination in advanced composites under variable-mode loading. *Mechanics of materials*, *38*(11), 1072-1089.

Camanho, P. P., Davila, C. G., & De Moura, M. F. (2003). Numerical simulation of mixed-mode progressive delamination in composite materials. *Journal of composite materials*, *37*(16), 1415-1438.

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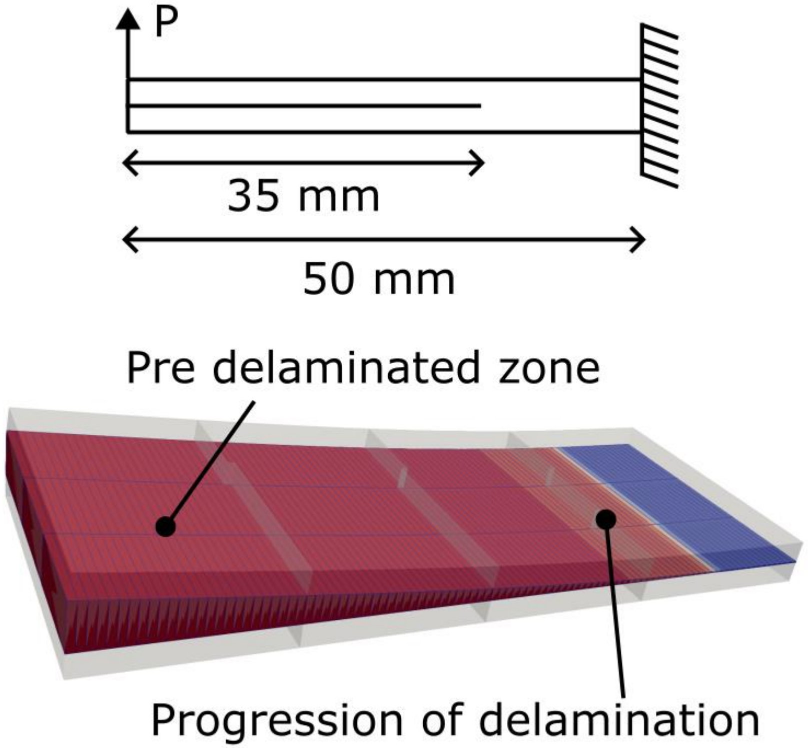

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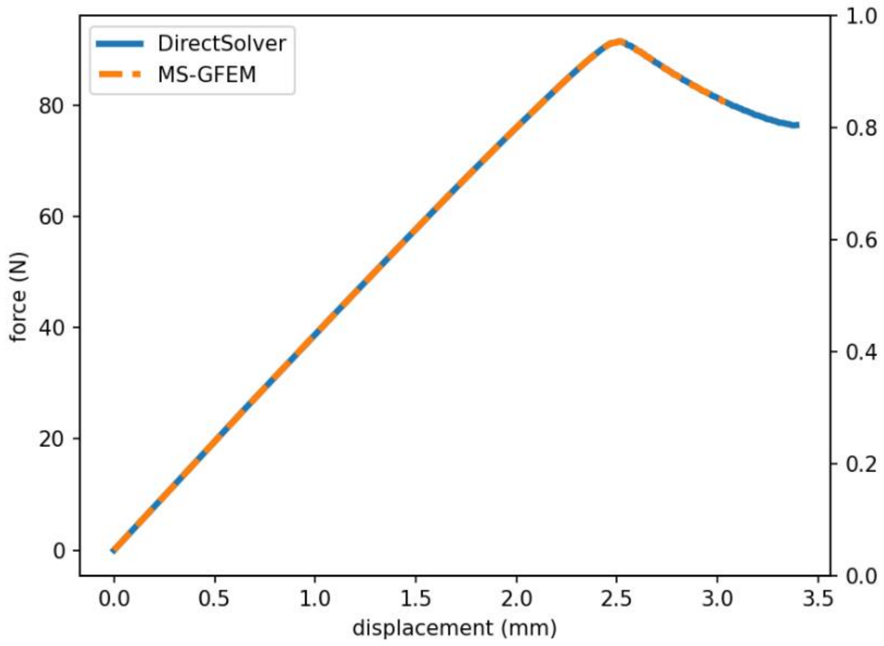

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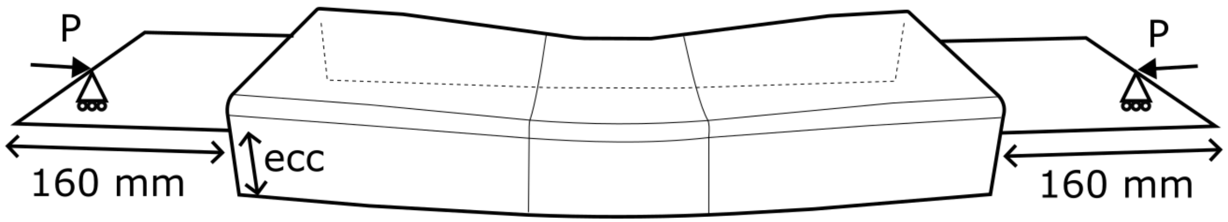
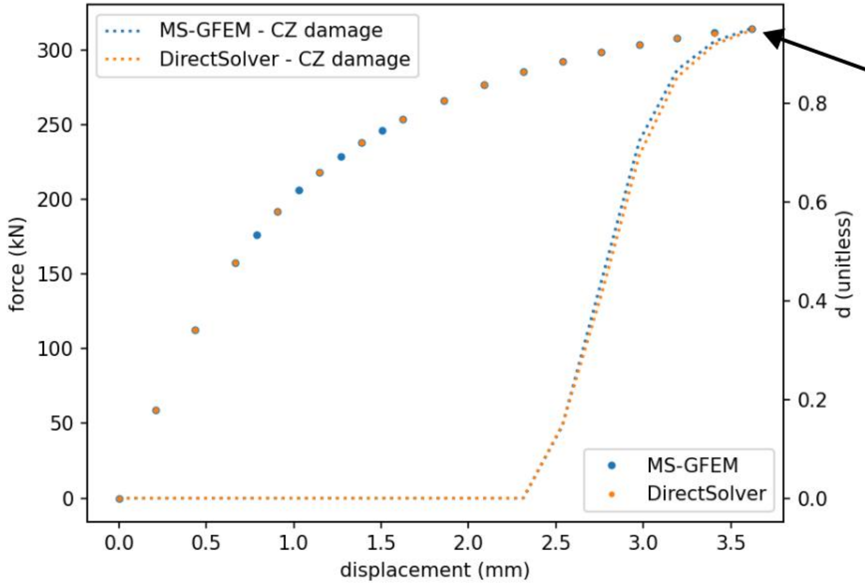
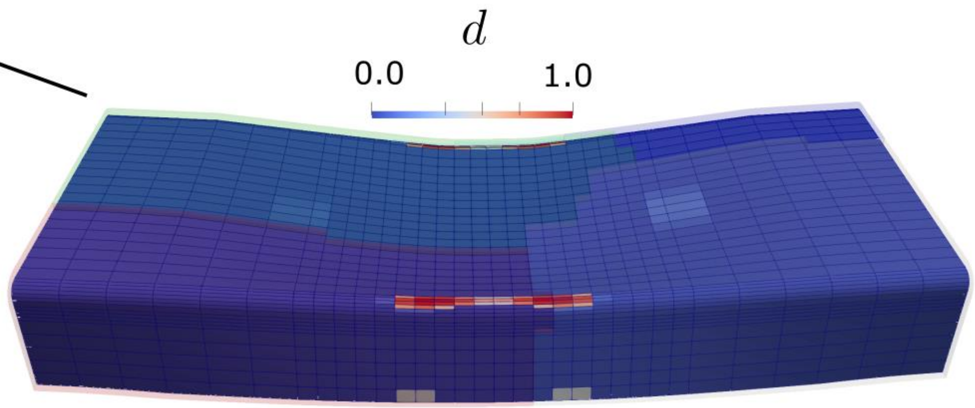
Cohesive element implementation

CerTest geometry — Cspar

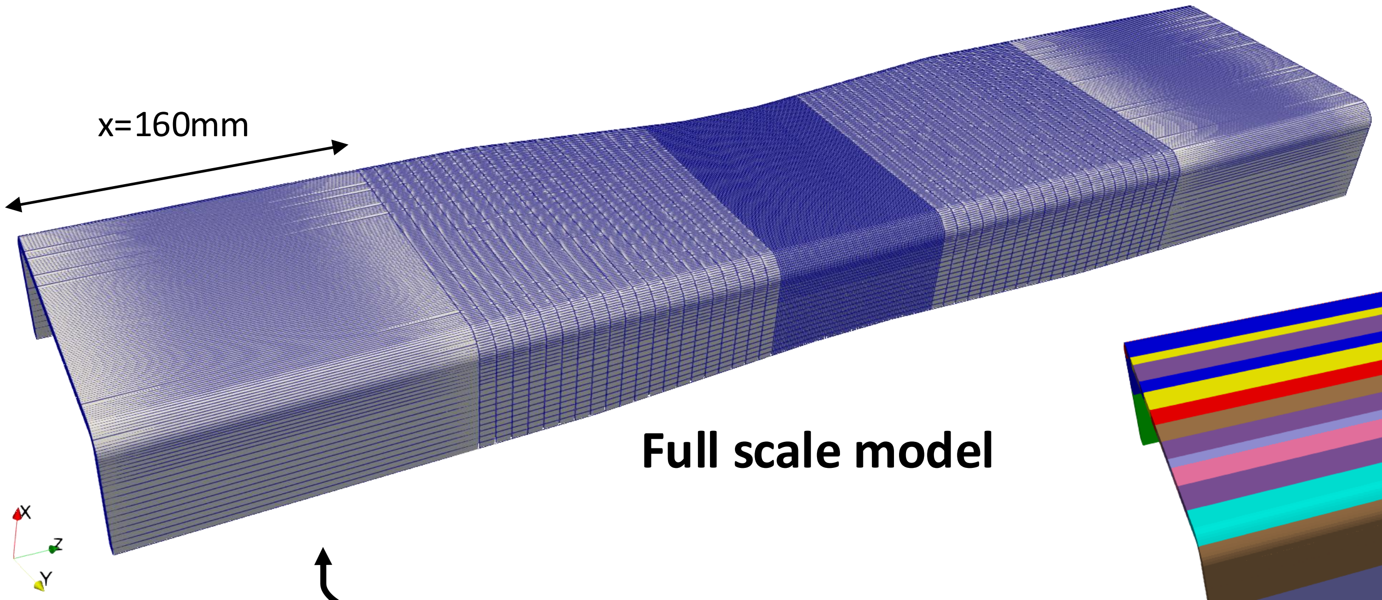






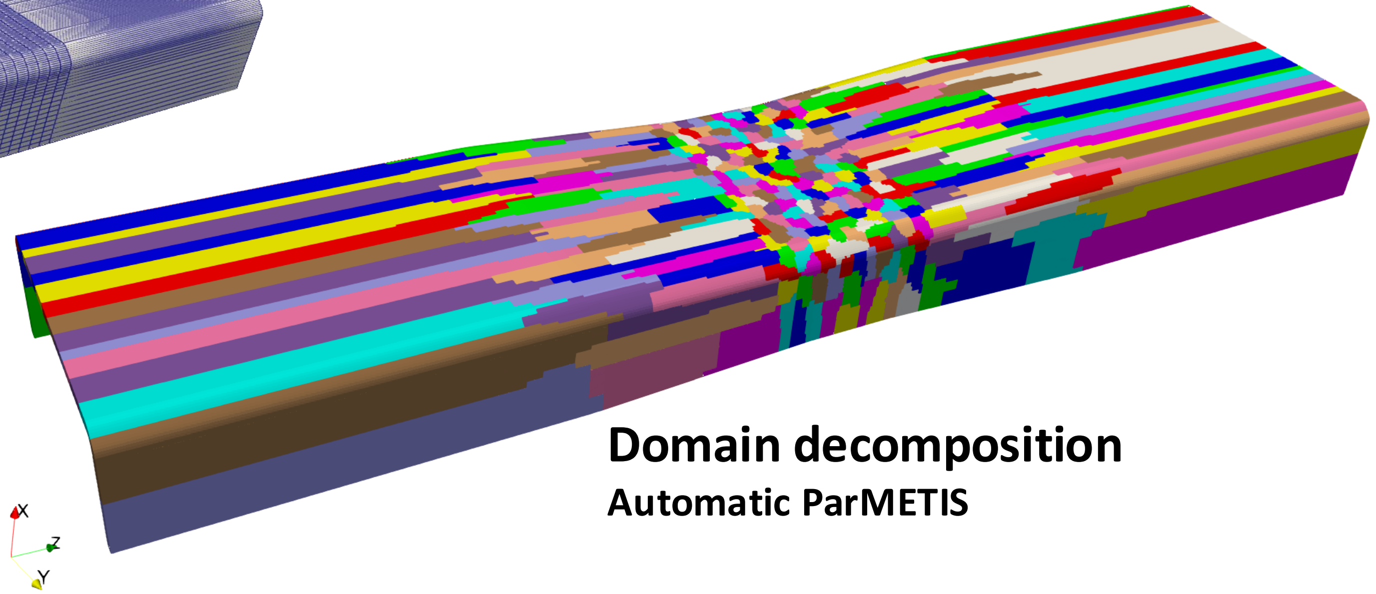
Turon, A., Davila, C. G., Camanho, P. P., & Costa, J. (2007). An engineering solution for mesh size effects in the simulation of delamination using cohesive zone models. *Engineering fracture mechanics*, *74*(10), 1665-1682.

Nonlinear MS-GFEM: Application to the CerTest demonstrator



Full scale model

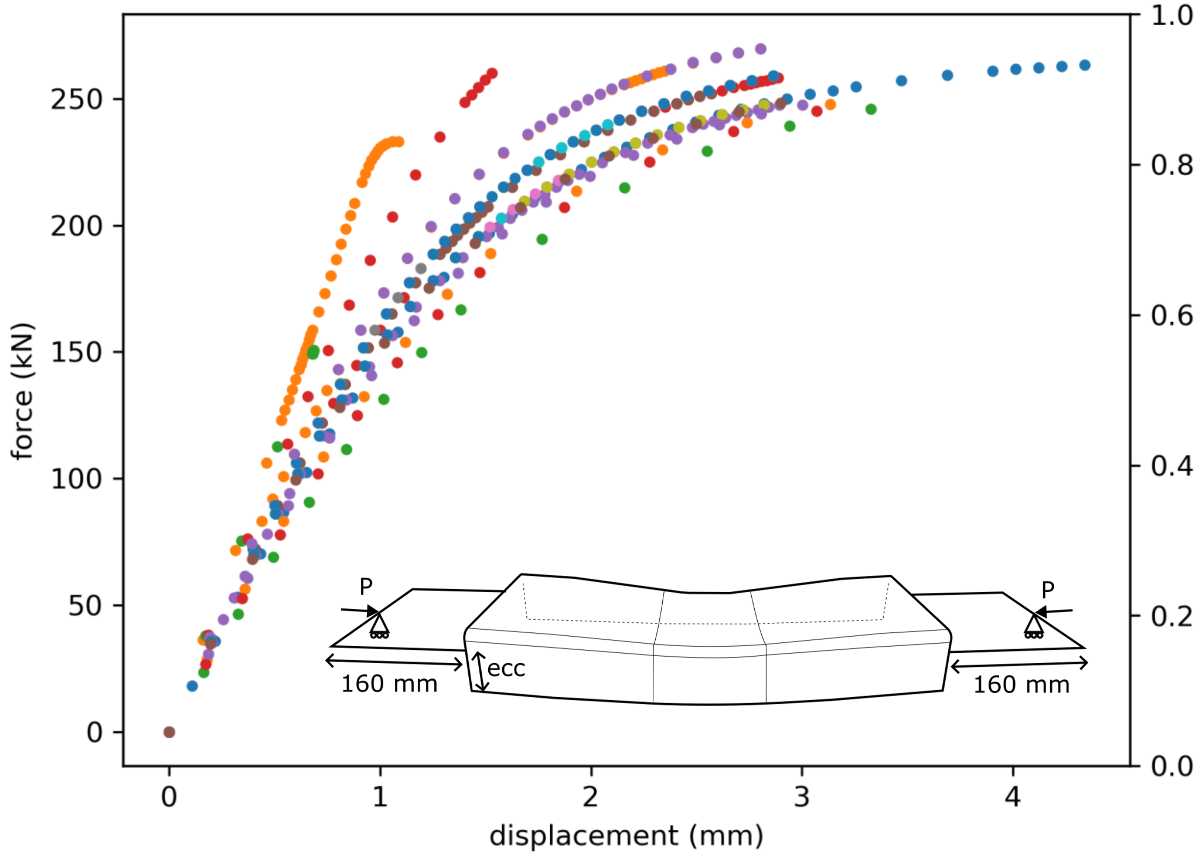
Boundary elements
Simply supported boundary condition



Domain decomposition
Automatic ParMETIS

- Full meso-scale → **2.1 Millions degrees of freedom** (linear elements)
- University of Durham HPC: Hamilton8
  - ➢ 120 standard compute nodes, each with 128 CPU cores (2x AMD EPYC 7702), 256GB RAM and 400GB local SSD storage.
- 320 processors used (4 nodes)
  - 7342 DoF per subdomain
  - 25,000 DoF in the coarse space
  - Model order reduction factor: **90**

Nonlinear MS-GFEM: Application to the CerTest demonstrator



Parameters:

Elastic properties: E_i G_{ij} υ_{ij}

Fracture properties: X_T X_C Y_T Y_C S_L Y_{3T} S_{13}

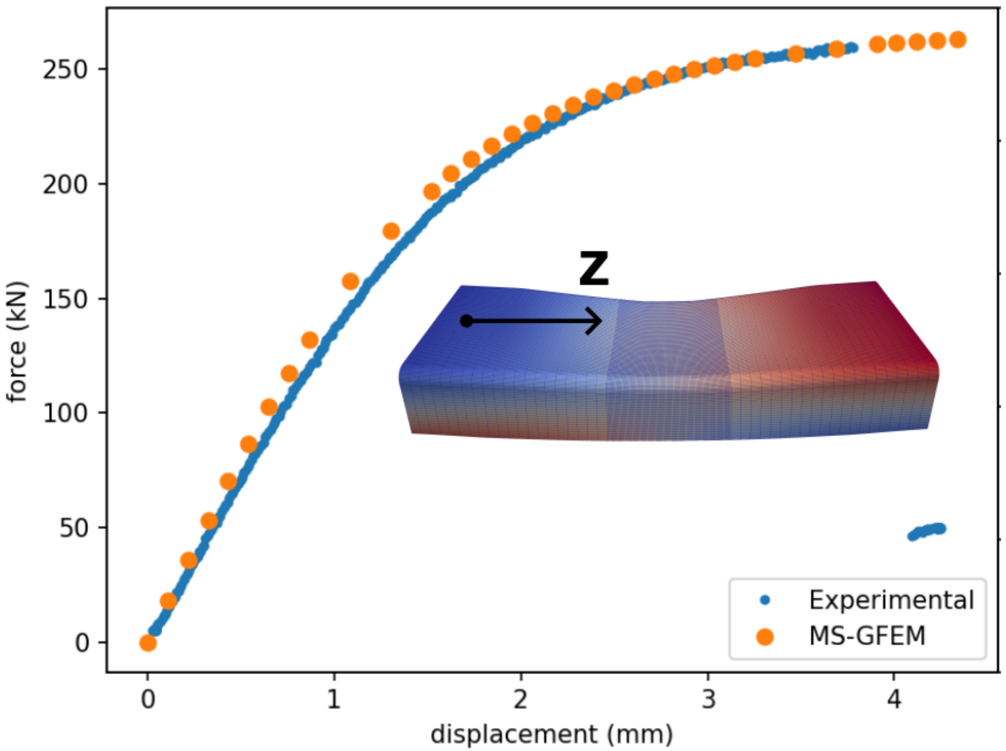
Cohesive properties: N_{cz} T_{cz} S_{cz} G_{IC} G_{IIC} G_{IIIC} BK,K_p

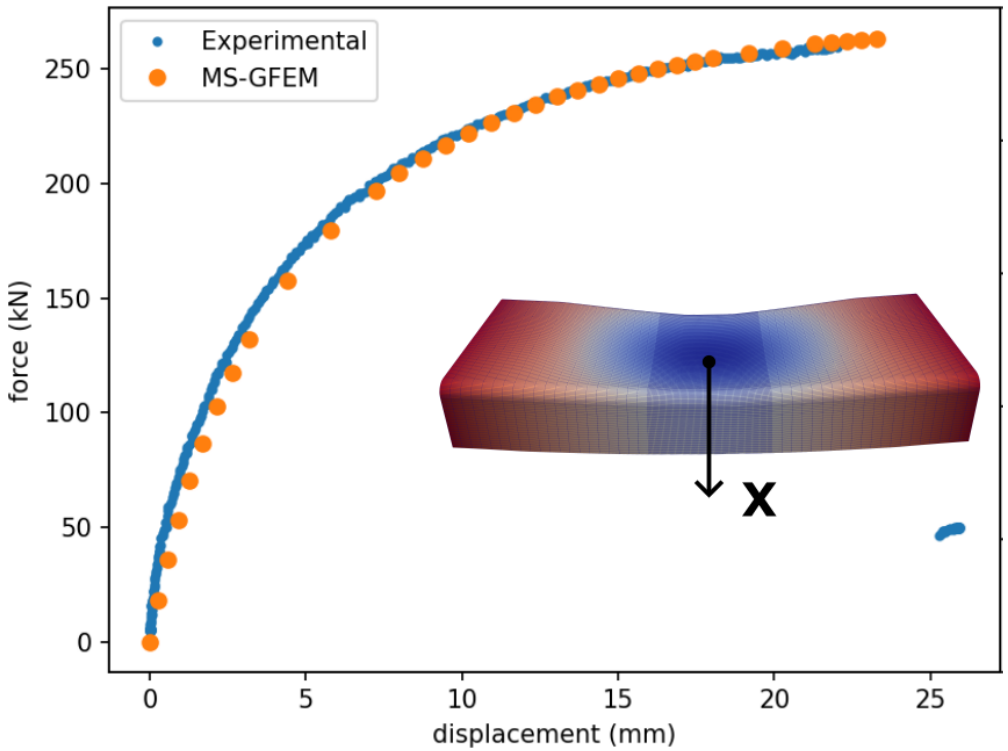
Boundary conditions: Ecc E_{rig}

Defect parameters: x_{delam} z_{delam} Interlayer

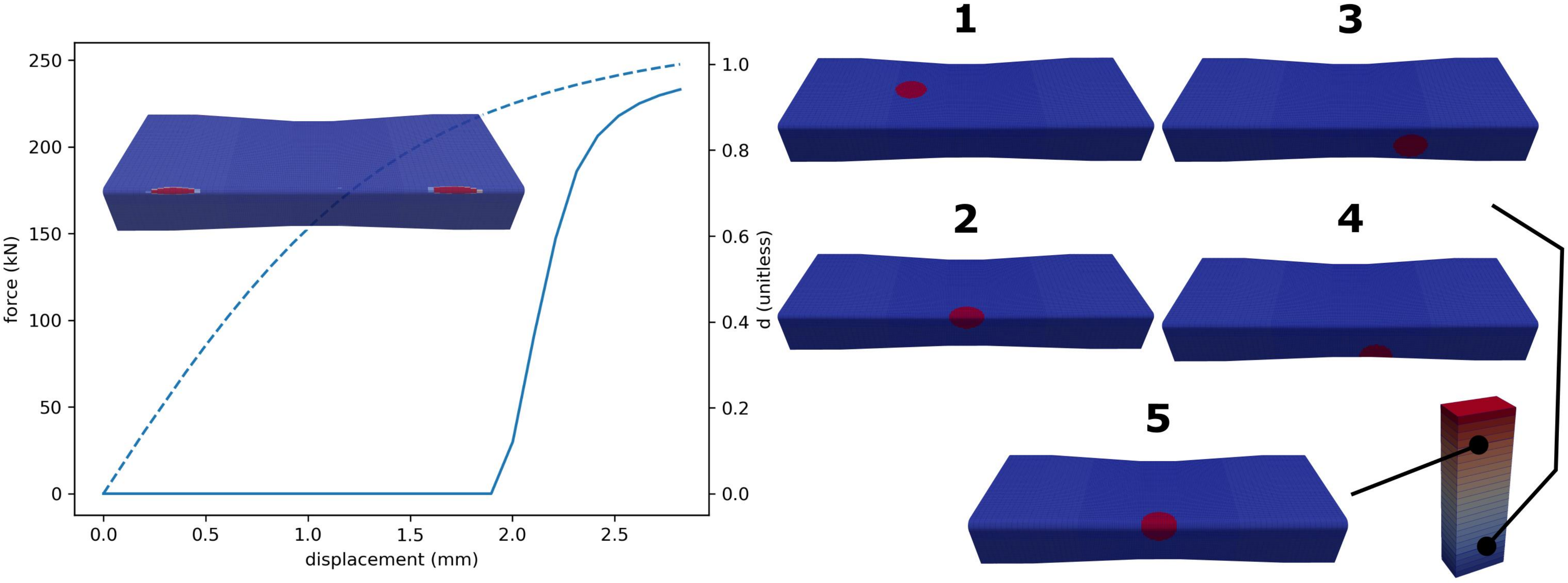
- **~ 4 minutes** per load increment (CS construction + Newton iterations)
- 1 to 3 hours on 320 processors HPC

Nonlinear MS-GFEM: Comparison with experimental DIC data





Nonlinear MS-GFEM: Exploration of the design space



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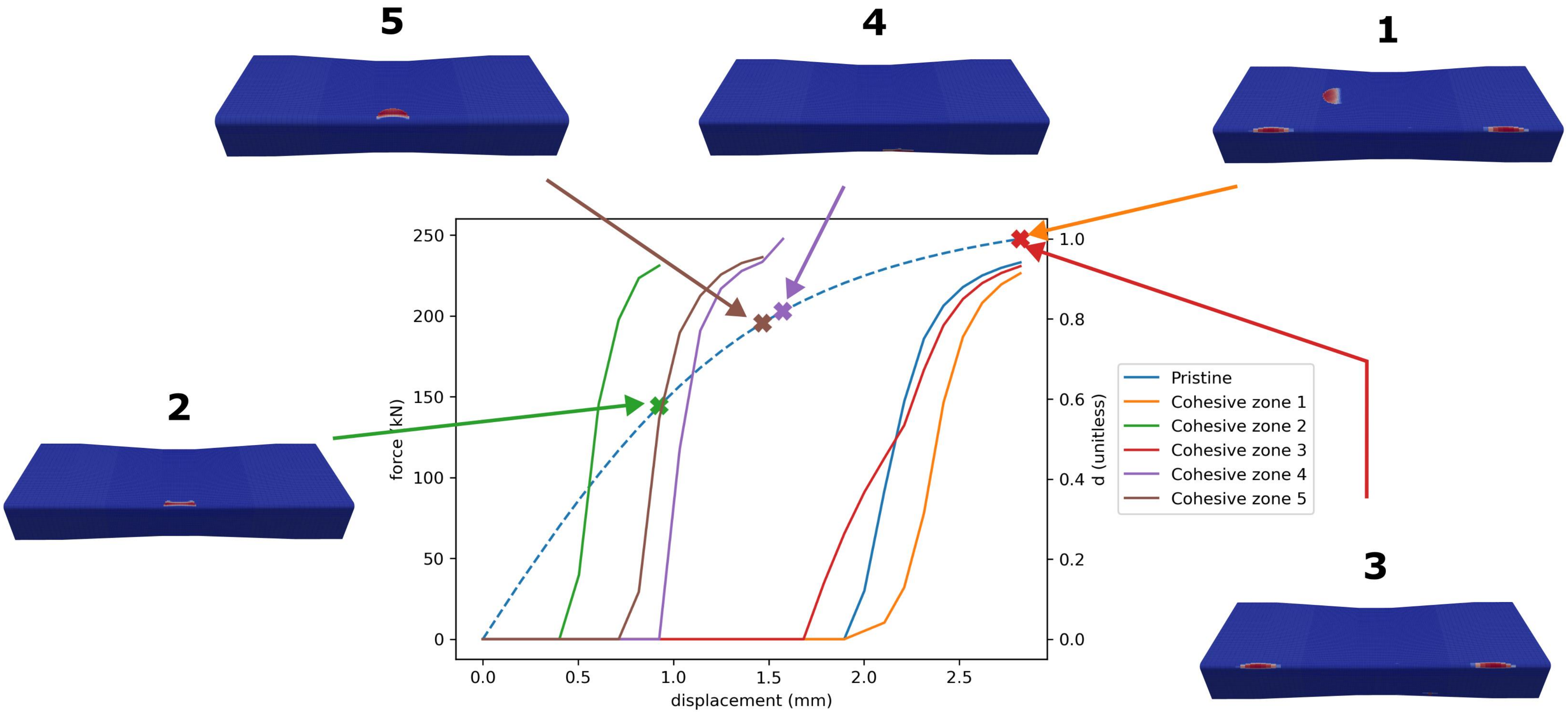
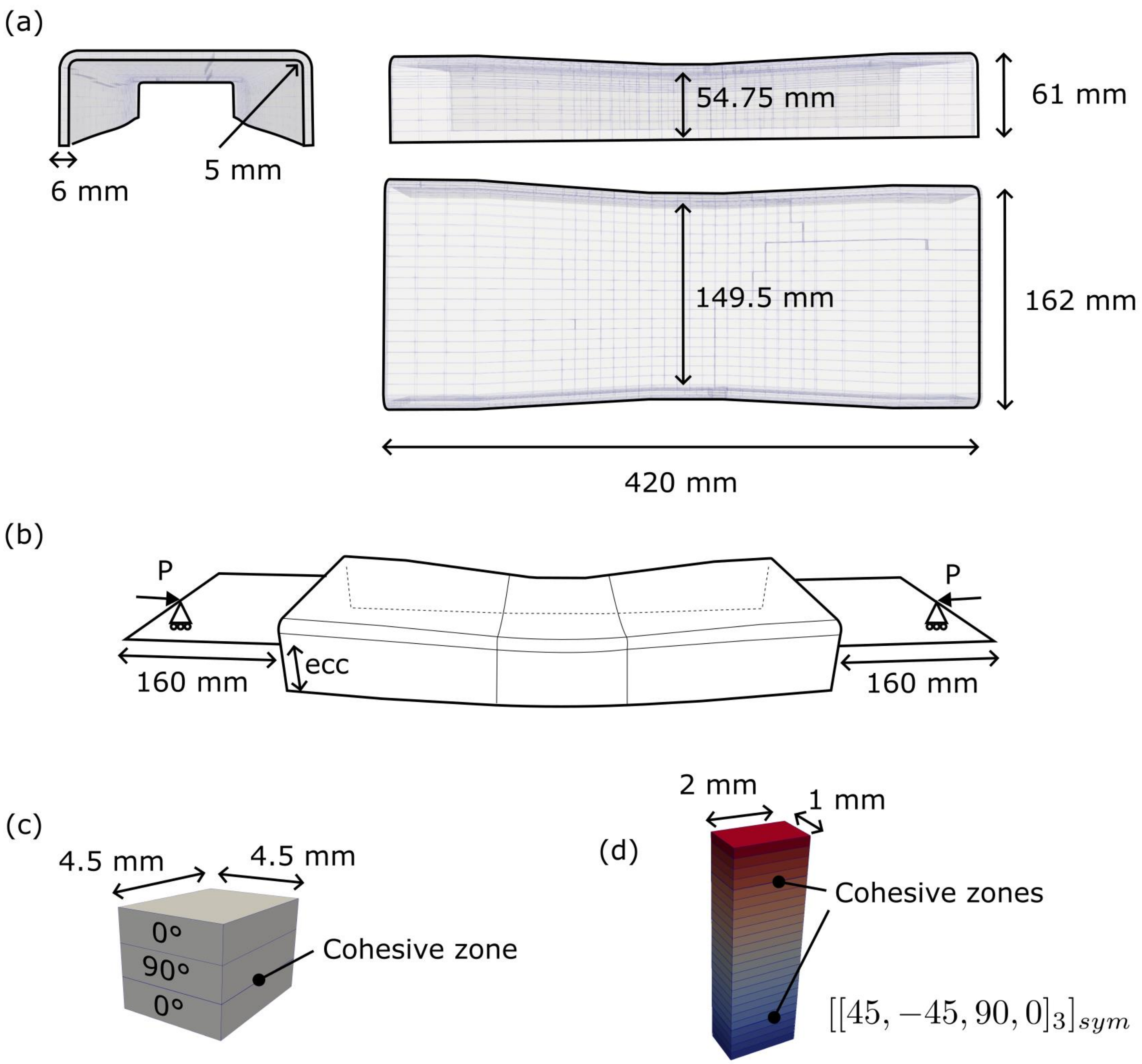


Conclusions

- Implementation & test of the **nonlinear MSGFEM**
- Very **efficient, parallel and scalable method** design for large-scale problem
  - ➢ Nonlinear solution for 2.1 M DoFs problem in 1-3 hours
- **No scale separation assumption**: **ANY defect** – region | shape | size
  - ➢ Independence on the domain decomposition
- Outlook: Spectral nonlinear approximation – Offline / Online


(a)
6 mm
5 mm
54.75 mm
61 mm
149.5 mm
162 mm
420 mm
(b)
P
P
ecc
160 mm
160 mm
(c)
4.5 mm
4.5 mm
0°
90°
0°
Cohesive zone
(d)
2 mm
1 mm
Cohesive zones
$[[45, -45, 90, 0]_3]_{sym}$