Bayesian Calibration of a Geometrically Nonlinear Finite Element C-spar Model using Digital Image Correlation

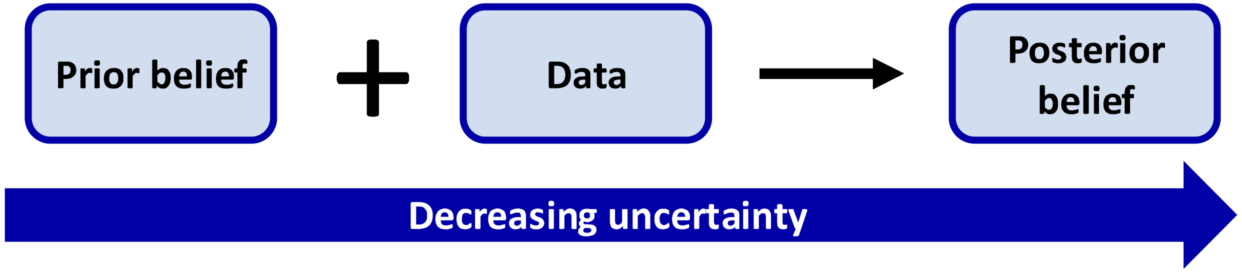
Carl Scarth[1], Geir Olafsson[2], Janice M. Dulieu-Barton[2],
Andrew T. Rhead[1], Richard Butler[1]

[1]Centre for Integrated Materials, Processes and Structures, University of Bath, Bath, UK
[2] Bristol Composites Institute, University of Bristol, Bristol, UK

21st European Conference on Composite Materials (ECCM 21), Nantes, France, 2nd July-5th July 2024

Background

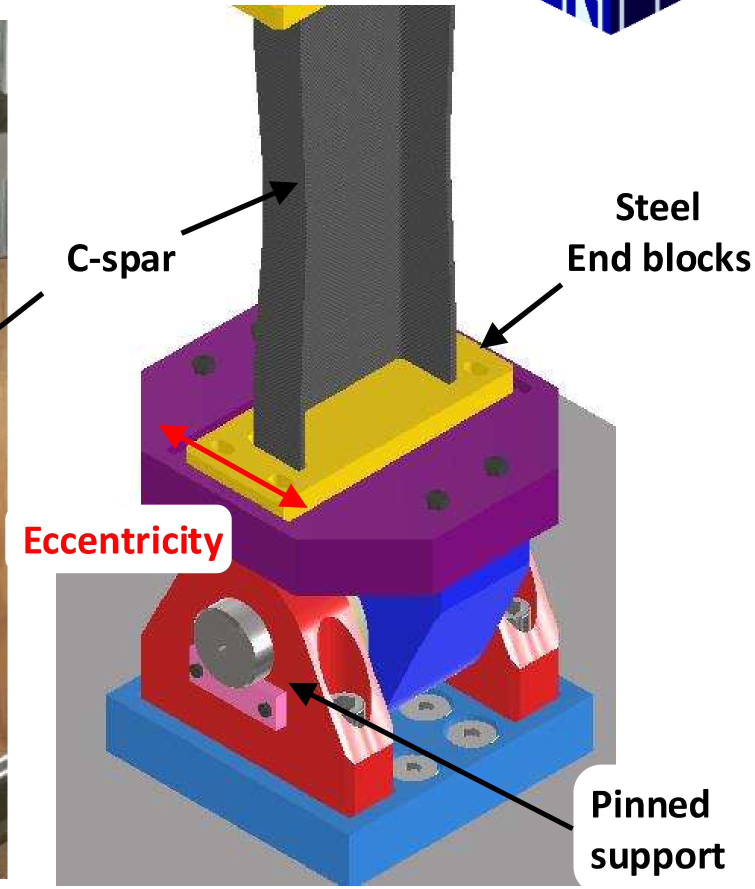
- All predictions made by engineers are subject to uncertainty:
  - Manufacturing uncertainty
  - How accurate are models?
  - How representative are physical tests of reality?
  - Unknown quantities e.g. internal defects/features, damage
- **Bayesian inference** helps us quantify, and reduce this uncertainty using new data



- **Calibration** is a way of using experimental data to inform future model predictions while accounting for unknown model inputs

Problem Specification

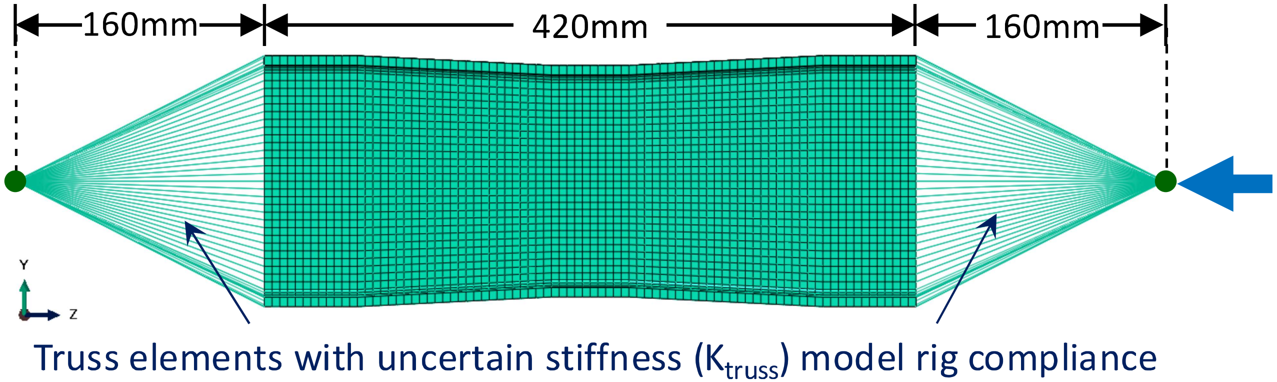
- Stereo DIC from Compression tests on a C-spar undertaken at University of Bristol using MatchID.
- Focus on longitudinal displacement on external corner.
- +4mm load eccentricity relative to gauge section centroid.
- ABAQUS model with material, boundary condition, and geometric uncertainty.
- **Can we use the test data to assess model accuracy given this uncertainty?**
- Main challenge is using large volumes of DIC data spanning both **time** and **space**, to calibrate **full-field** model output.

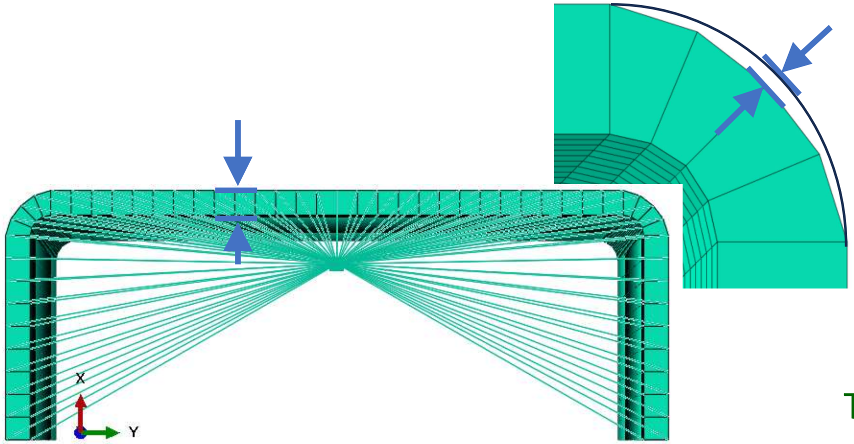


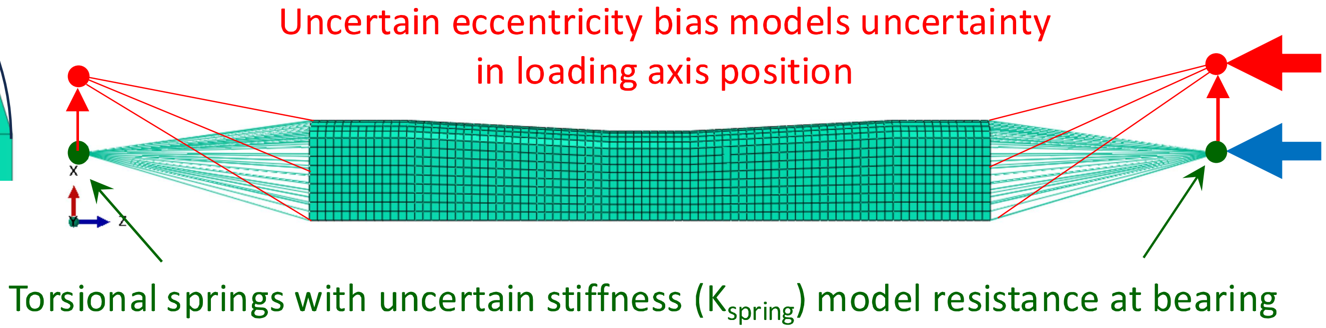
Test fixture design showing boundary conditions

ABAQUS Model and Uncertainties

- Modelled with 4536 SC8R continuum shell elements (9354 nodes).
- Simply supported at reference nodes offset from spar.
- Uncertainty in boundary conditions.
- Geometric uncertainty in (ply) thickness and corner thinning.
- Uncertainty in longitudinal modulus E_{11}.



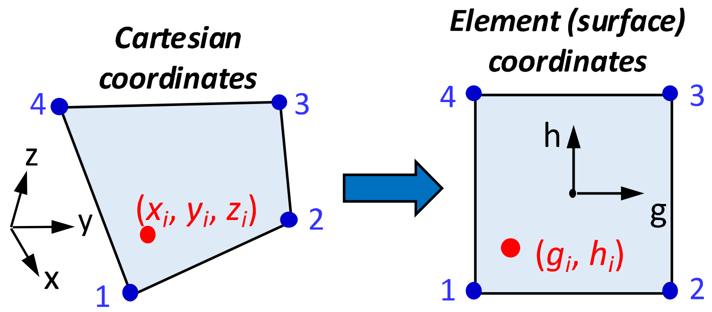


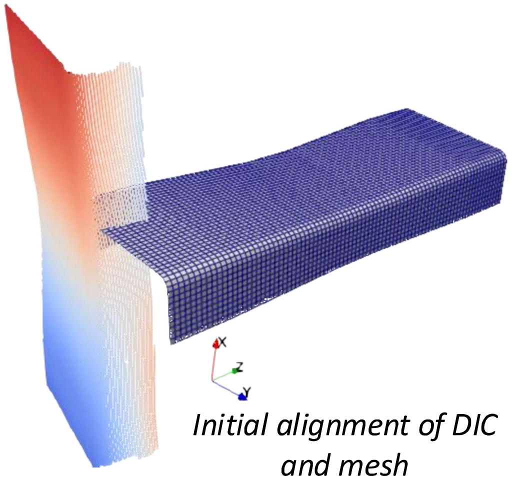


Alignment of DIC and model

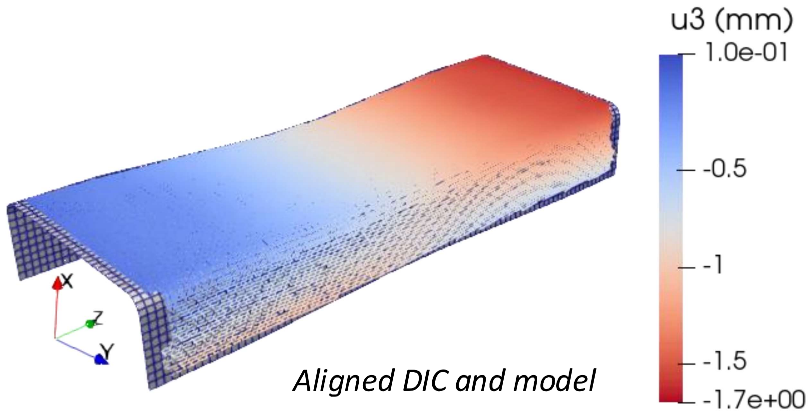
Set of tools developed (or adopted) for mapping DIC onto an ABAQUS mesh and making pointwise comparisons:

1) Point-cloud/mesh registration in CloudCompare[1].
2) Newton-Raphson method to transform from Cartesian coordinates to local element coordinates.
3) Interpolation across spatial domain using ABAQUS shape functions.
4) Linear interpolation across time domain to match solver increments.

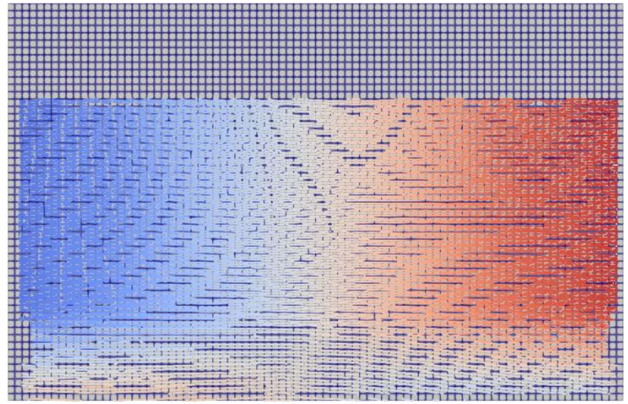




Initial alignment of DIC and mesh



Aligned DIC and model



DIC data transformed into element coordinates, arranged in regular grid.

[1] CloudCompare Version 2.11.3 [GPL Software], ttps://www.cloudcompare.org/

Calibration: Methodology

- Following Higdon et al.[2], method for calibrating models with vector-valued output:

$$\underbrace{\boldsymbol{y}}_{\text{DIC point cloud}} = \underbrace{\boldsymbol{\eta}(\boldsymbol{\theta}^*)}_{\text{Abaqus model}} + \underbrace{\boldsymbol{\varepsilon}}_{\text{Error}}$$

$\boldsymbol{\theta}^*$ = uncertain inputs we want to learn about:

E_{11}, ply thickness, eccentricity bias, K_{truss}, radius thinning, K_{spring}

- An inverse problem:
  - $\boldsymbol{y}$ is known.
  - $\boldsymbol{\theta}^*$ and magnitude of error $\boldsymbol{\varepsilon}$ are uncertain.
  - Gaussian process emulator used as surrogate for Abaqus, $\boldsymbol{\eta}$, which also has uncertain parameters.
- Infer values for uncertain parameters by first specifying **prior distributions**.
- Then sample from the **posterior distribution** using Hamiltonian Monte Carlo (No-U-Turn Sampler, NUTS) in Stan[3].

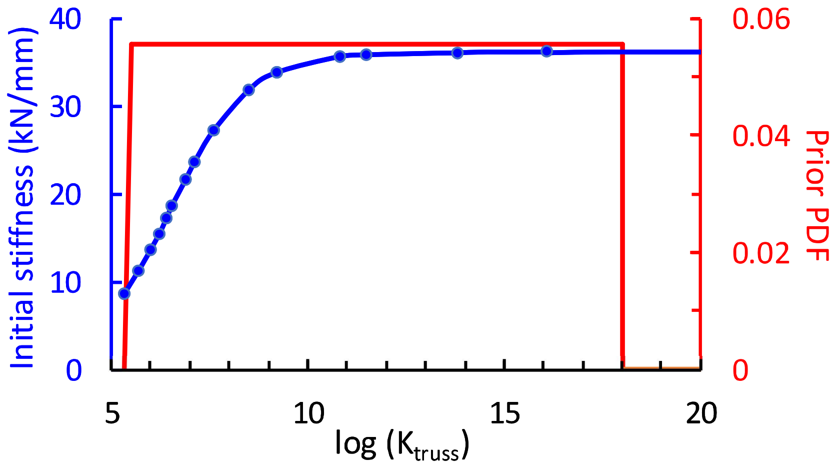
[2] *D. Higdon et al, "Computer model calibration using high-dimensional output", Journal of the American Statistical Association, 2008*
[3] *Stan modeling language users guide and reference manual, Version 2.26.1, https://mc-stan.org*

Calibration: Prior distributions

Prior parameters: mean and standard deviation for Gaussian and Half-normal, upper and lower bounds for uniform

	E_{11} (GPa)	t_{ply} (mm)	Eccentricity bias (mm)	$\log(K_{truss})$	Radius thinning (mm)	$\log(K_{spring})$
Distribution	Gaussian	Gaussian	Gaussian	Uniform	Half-normal	Half-normal
Parameter 1	140.9	0.125	0.0	5.5	0.0	0.0
Parameter 2	8.454	0.005	0.667	18.0	0.667	2.0

- What do I **believe** about the uncertain inputs before the test?
- **Not a measure of variability**
- Rig stiffness, K_{truss} based on parametric study of initial gradient of force-displacement curves.
- Compressive modulus E_{11} taken from published coupon test data[4]. Hexcel state 150GPa[5]. *Which value is correct*?



Initial gradient of force-displacement curve at reference node, vs log of rig stiffness

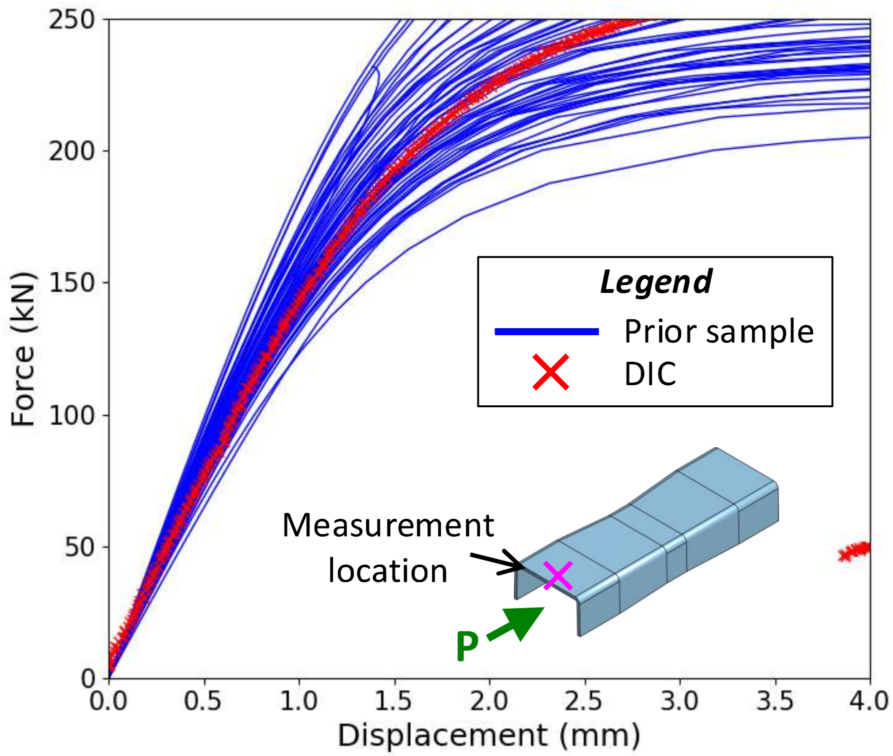
[4] *E. Clarkson. "Hexcel 8552 IM7 unidirectional prepreg 190 gsm & 35% qualification statistical analysis report." NCAMP, 2019.*
[5] *HexTow® IM7 Carbon Fibre, Product Data Sheet, HEXCEL*

Calibration: Emulator for ABAQUS

- Surrogate model fitted to ABAQUS output from 60 Latin Hypercube samples across input prior distributions.
- Decompose output into $p = 16$ principal components via SVD[2]:

$$\boldsymbol{\eta}(\boldsymbol{\theta}^*) \approx \sum_{i=1}^{p} \boldsymbol{\phi}_i(\boldsymbol{x}, t)\, w_i(\boldsymbol{\theta}^*)$$

- Basis vectors $\boldsymbol{\phi}_i$ capture spatial and temporal dependency.



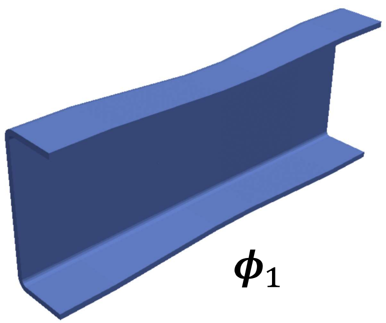
Force vs displacement at point on web for training samples and DIC

Calibration: Emulator for ABAQUS

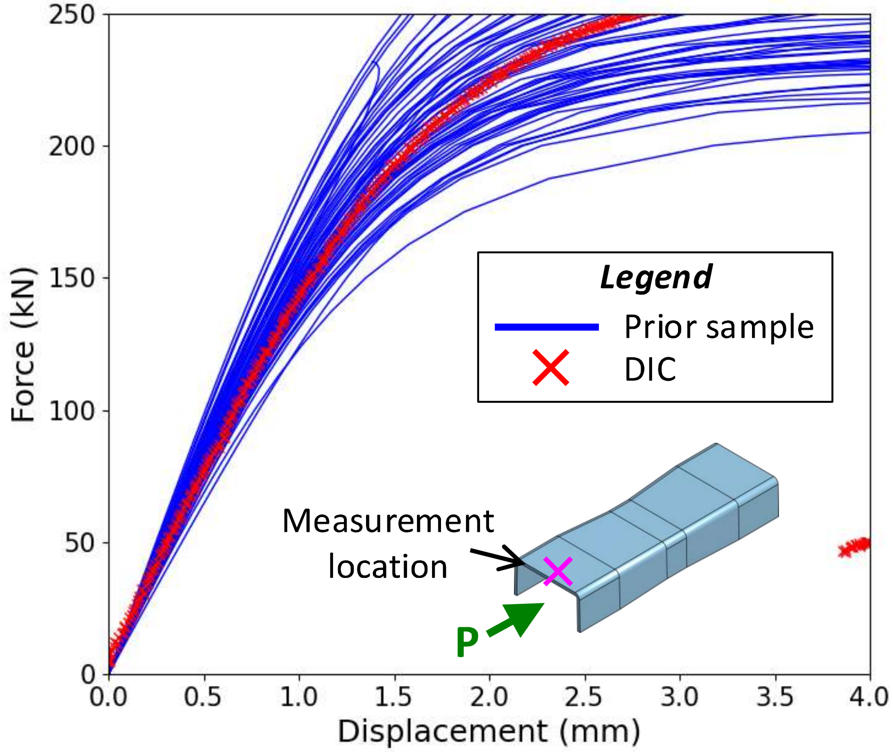
- Surrogate model fitted to ABAQUS output from 60 Latin Hypercube samples across input prior distributions.
- Decompose output into $p = 16$ principal components via SVD[4]:

$$\boldsymbol{\eta}(\boldsymbol{\theta}^*) \approx \sum_{i=1}^{p} \boldsymbol{\phi}_i(\boldsymbol{x}, t)\, w_i(\boldsymbol{\theta}^*)$$

- Basis vectors $\boldsymbol{\phi}_i$ capture spatial and temporal dependency.



First three basis vectors for emulator



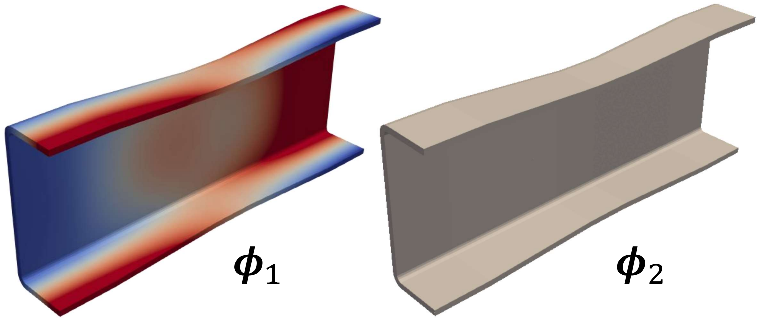
Force vs displacement at point on web for training samples and DIC

Calibration: Emulator for ABAQUS

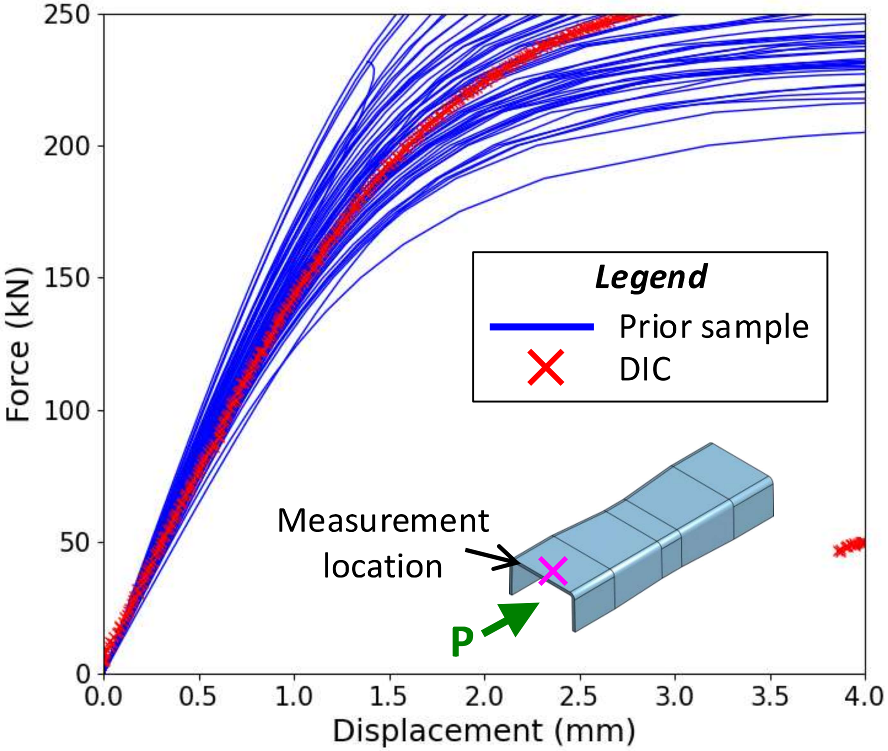
- Surrogate model fitted to ABAQUS output from 60 Latin Hypercube samples across input prior distributions.
- Decompose output into $p = 16$ principal components via SVD[4]:

$$\boldsymbol{\eta}(\boldsymbol{\theta}^*) \approx \sum_{i=1}^{p} \boldsymbol{\phi}_i(\boldsymbol{x}, t)\, w_i(\boldsymbol{\theta}^*)$$

- Basis vectors $\boldsymbol{\phi}_i$ capture spatial and temporal dependency.



First three basis vectors for emulator



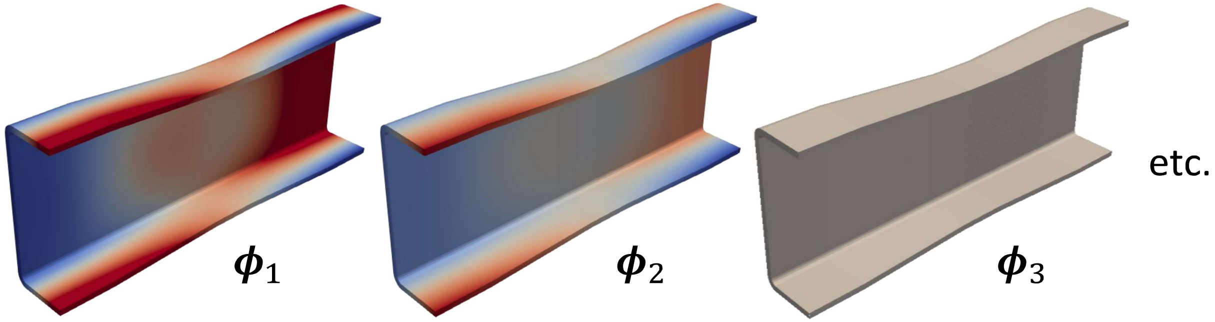
Force vs displacement at point on web for training samples and DIC

Calibration: Emulator for ABAQUS

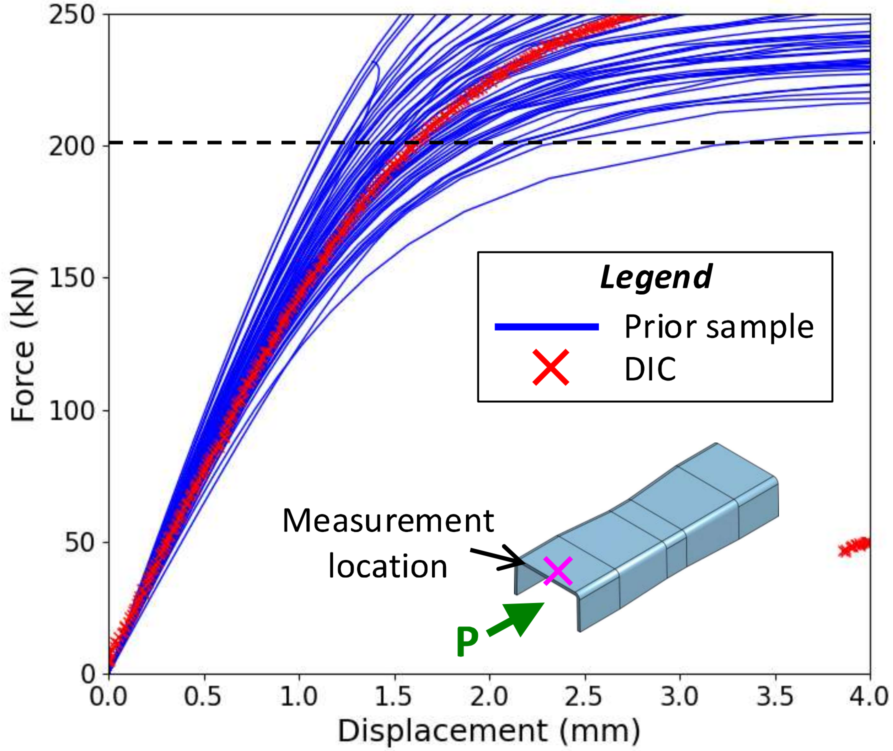
- Surrogate model fitted to ABAQUS output from 60 Latin Hypercube samples across input prior distributions.
- Decompose output into $p = 16$ principal components via SVD[4]:

$$\boldsymbol{\eta}(\boldsymbol{\theta}^*) \approx \sum_{i=1}^{p} \boldsymbol{\phi}_i(\boldsymbol{x}, t)\, w_i(\boldsymbol{\theta}^*)$$

- Basis vectors $\boldsymbol{\phi}_i$ capture spatial and temporal dependency.
- Gaussian process emulators w_i model uncertain input dependency.
- Need to truncate output to 200kN so solver converges at all increments for all training samples.

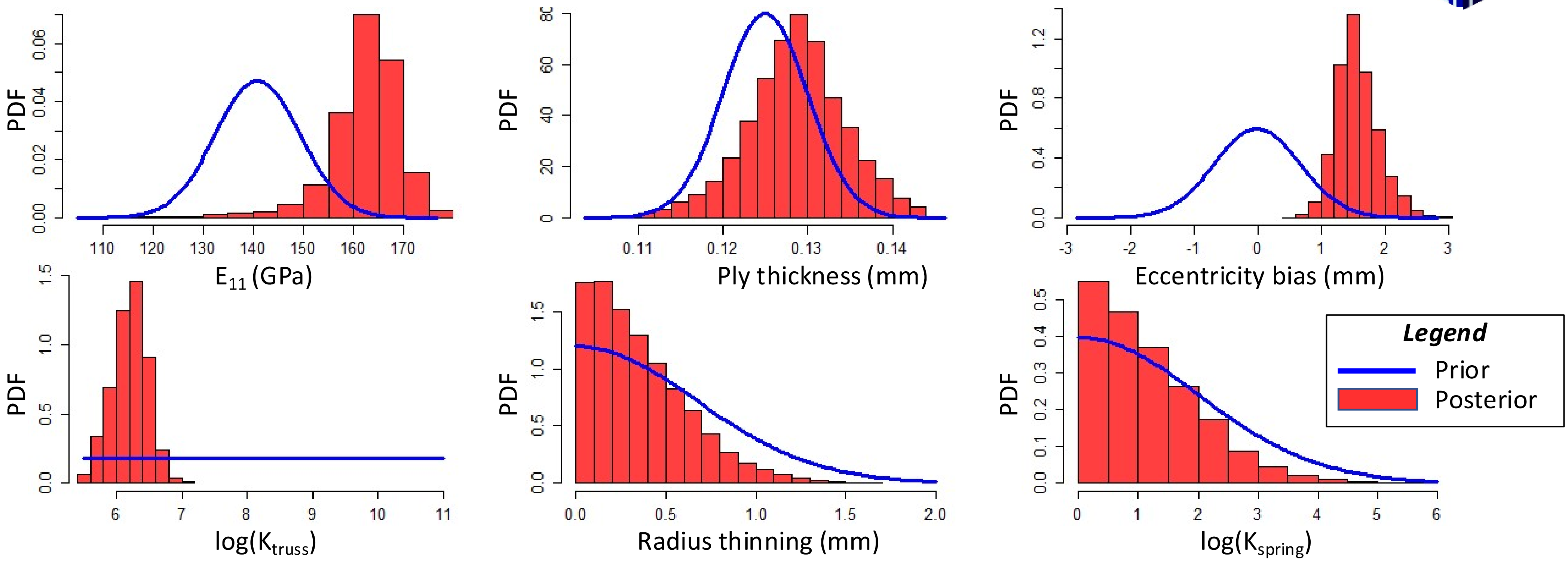


First three basis vectors for emulator

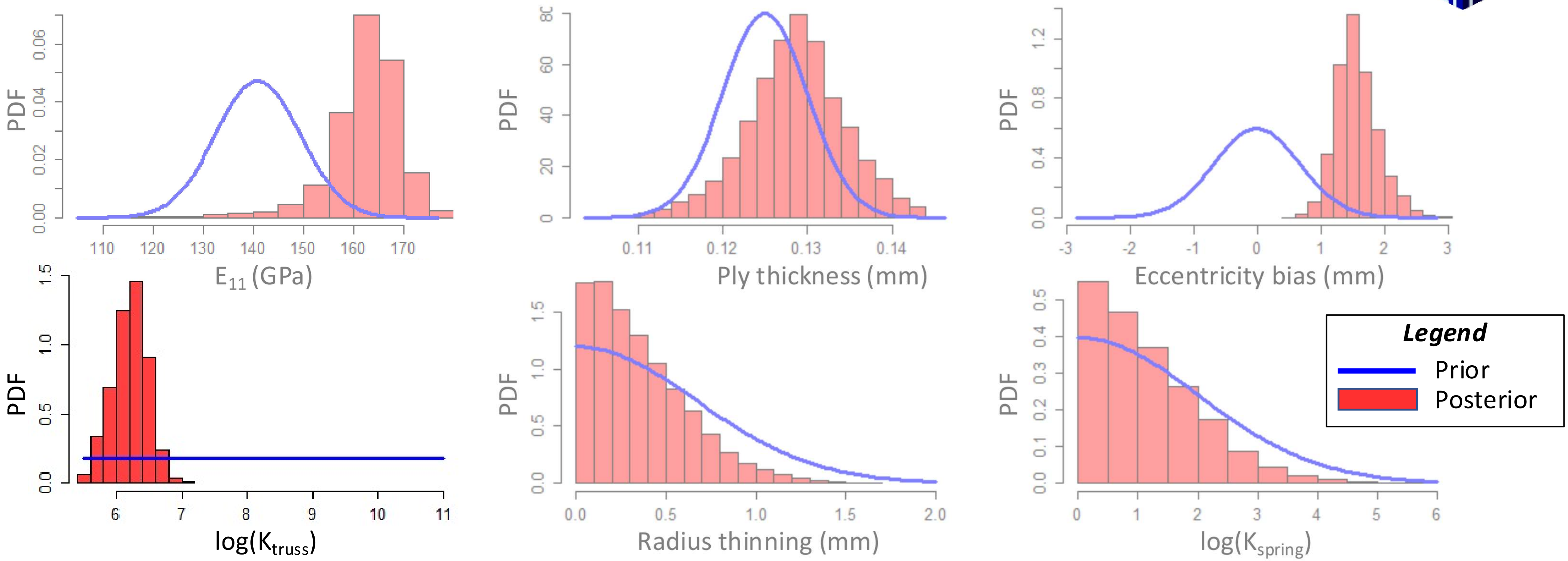


Force vs displacement at point on web for training samples and DIC

Results: Posterior distributions

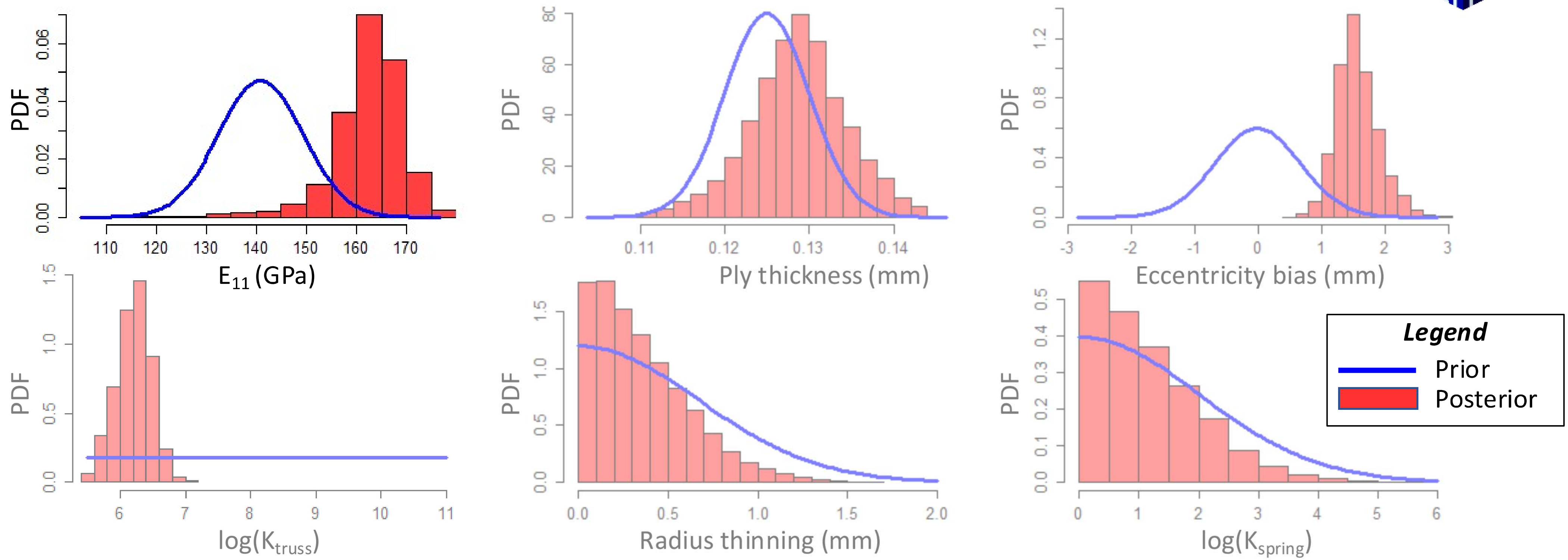


Results: Posterior distributions



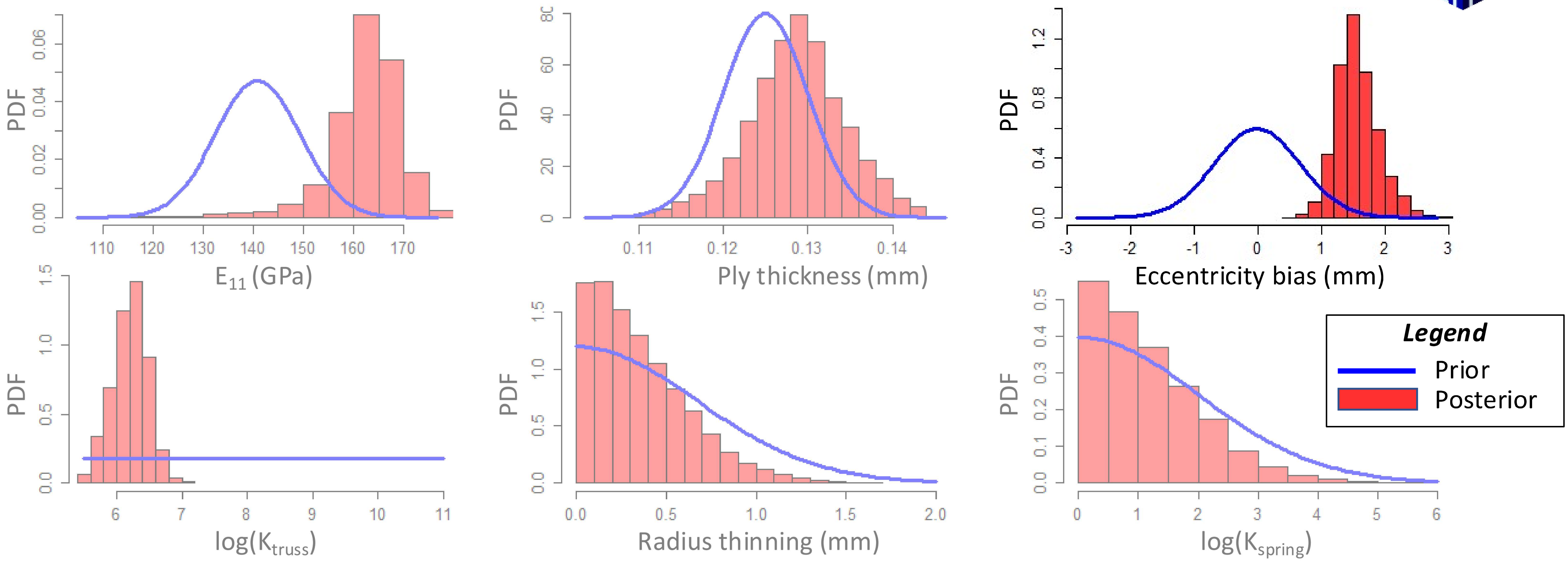
- K_{truss} shifts to very low values (approx. 40% knockdown in overall stiffness) indicating significant rig or machine compliance.

Results: Posterior distributions



- K_{truss} shifts to very low values (approx. 40% knockdown in overall stiffness) indicating significant rig or machine compliance.
- E_{11} shifts towards higher values, with mode of 164 GPa similar to tensile modulus.

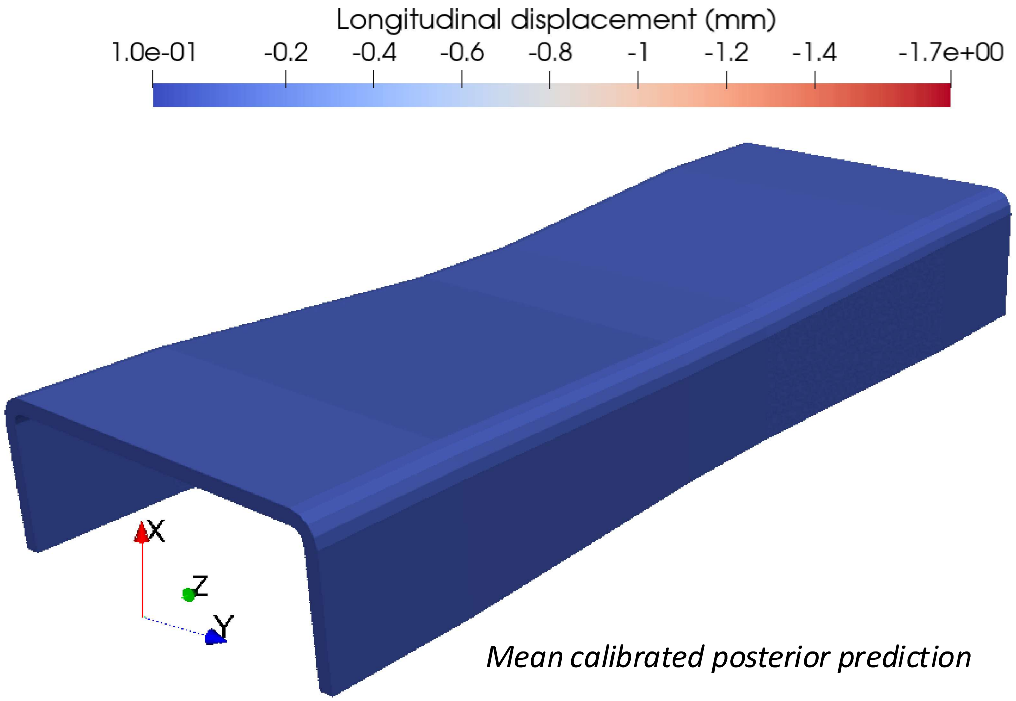
Results: Posterior distributions



- K_{truss} shifts to very low values (approx. 40% knockdown in overall stiffness) indicating significant rig or machine compliance.
- E_{11} shifts towards higher values, with mode of 164 GPa similar to tensile modulus.
- Strong shift of loading axis via eccentricity bias, which later highlighted a manufacturing error.

Results: Calibrated predictions

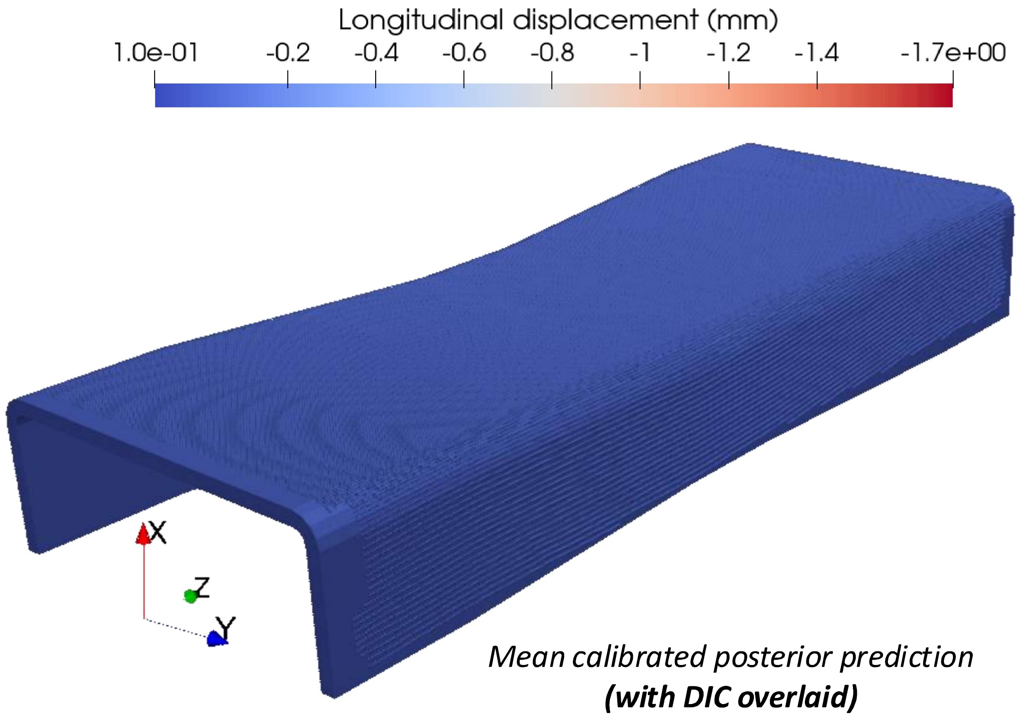
- Run posterior samples through emulator and average out uncertainty to get prediction.



Mean calibrated posterior prediction

Results: Calibrated predictions

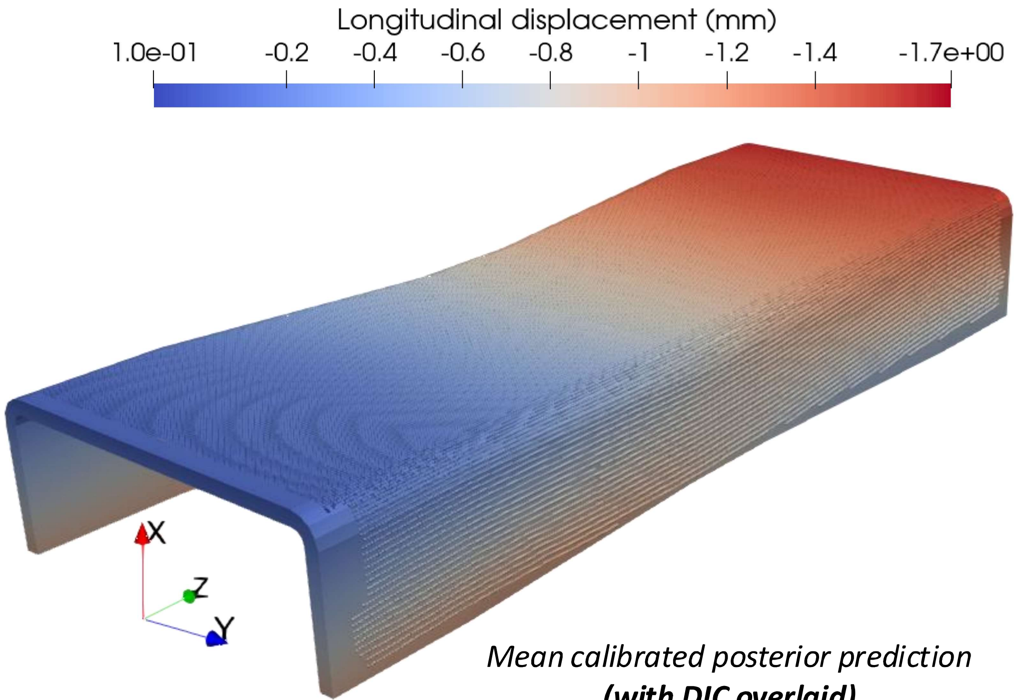
- Run posterior samples through emulator and average out uncertainty to get prediction.



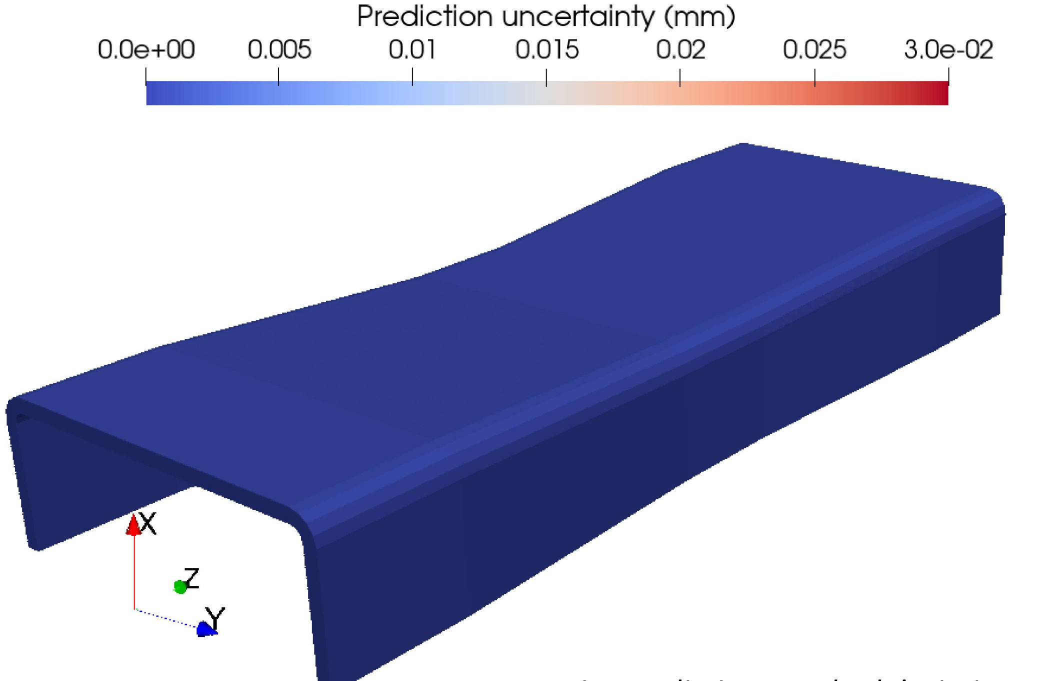
Mean calibrated posterior prediction
(with DIC overlaid)

Results: Calibrated predictions

- Run posterior samples through emulator and average out uncertainty to get prediction.
- Standard deviation indicates regions of highest posterior uncertainty.



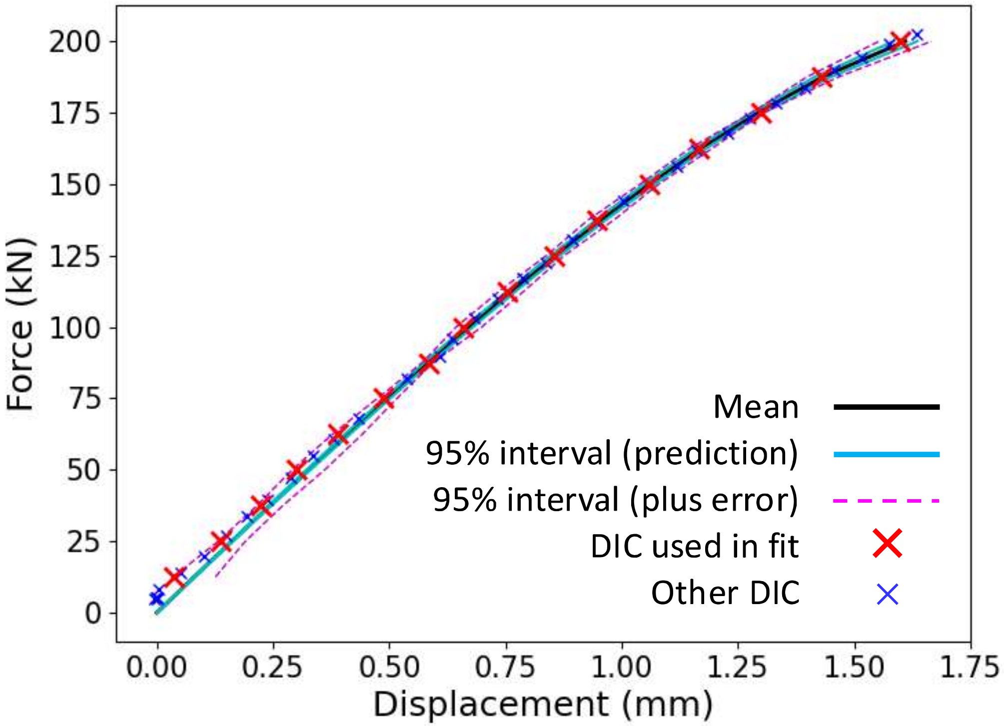
Mean calibrated posterior prediction
(with DIC overlaid)



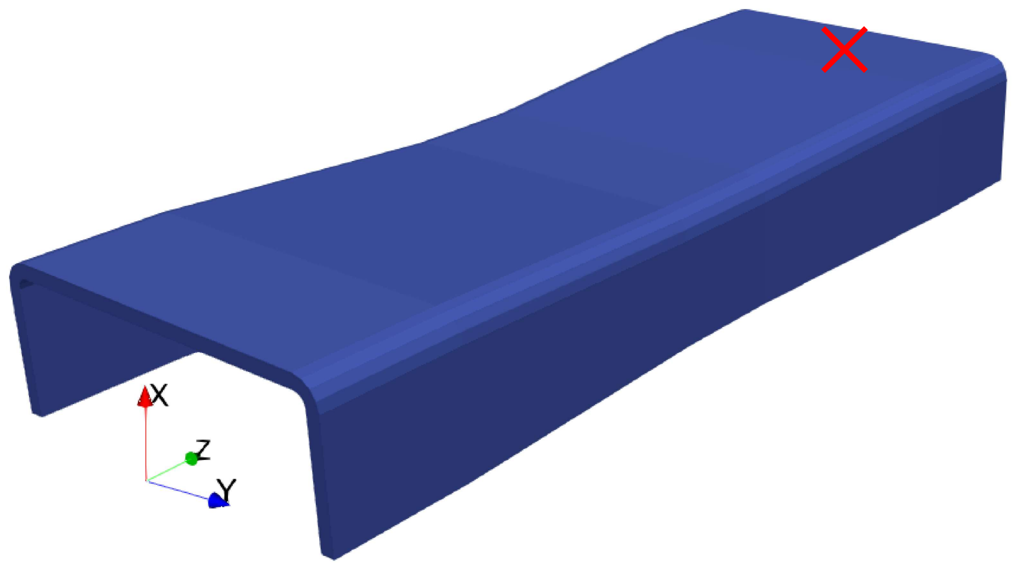
Posterior prediction standard deviation

Results: Residuals with data

- Extract force-displacement at fixed location to visualise fit

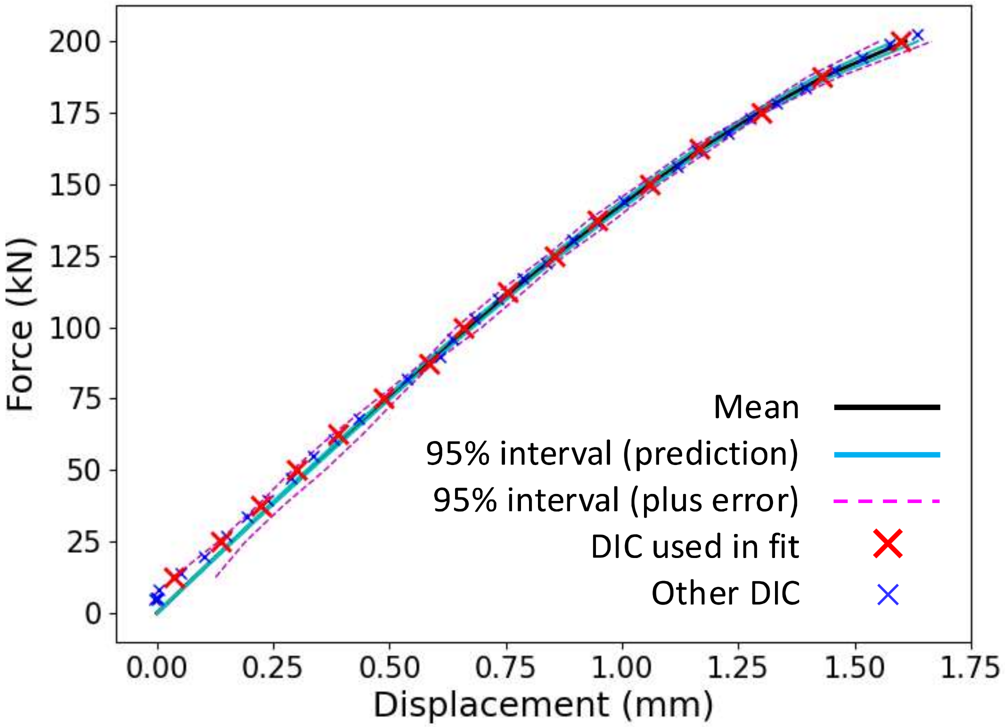


Posterior prediction for point (X) on web

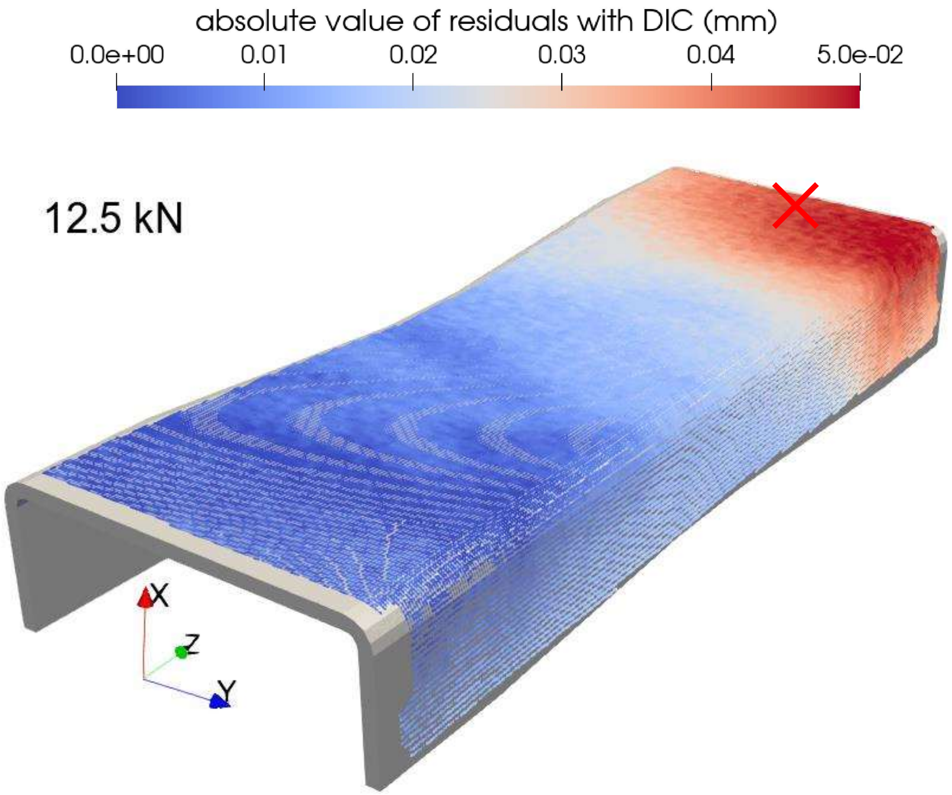


Results: Residuals with data

- Extract force-displacement at fixed location to visualise fit
- Subtract the DIC from each increment of calibrated model to visualise residuals highlighting regions of discrepancy.



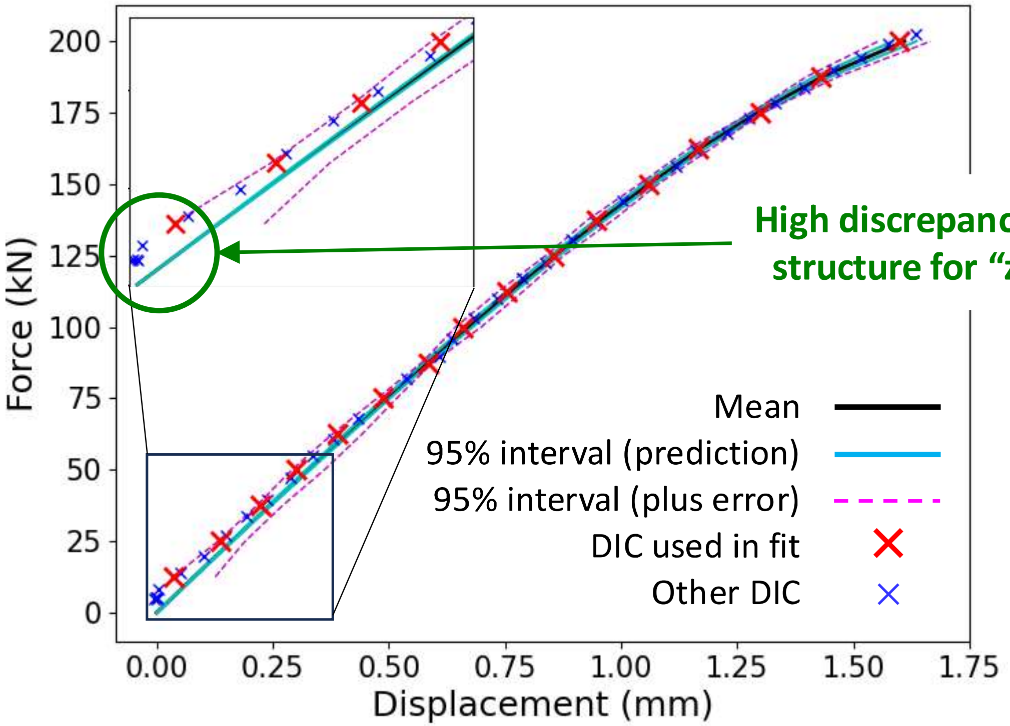
Posterior prediction for point (X) on web



Absolute residual of mean prediction compared with DIC

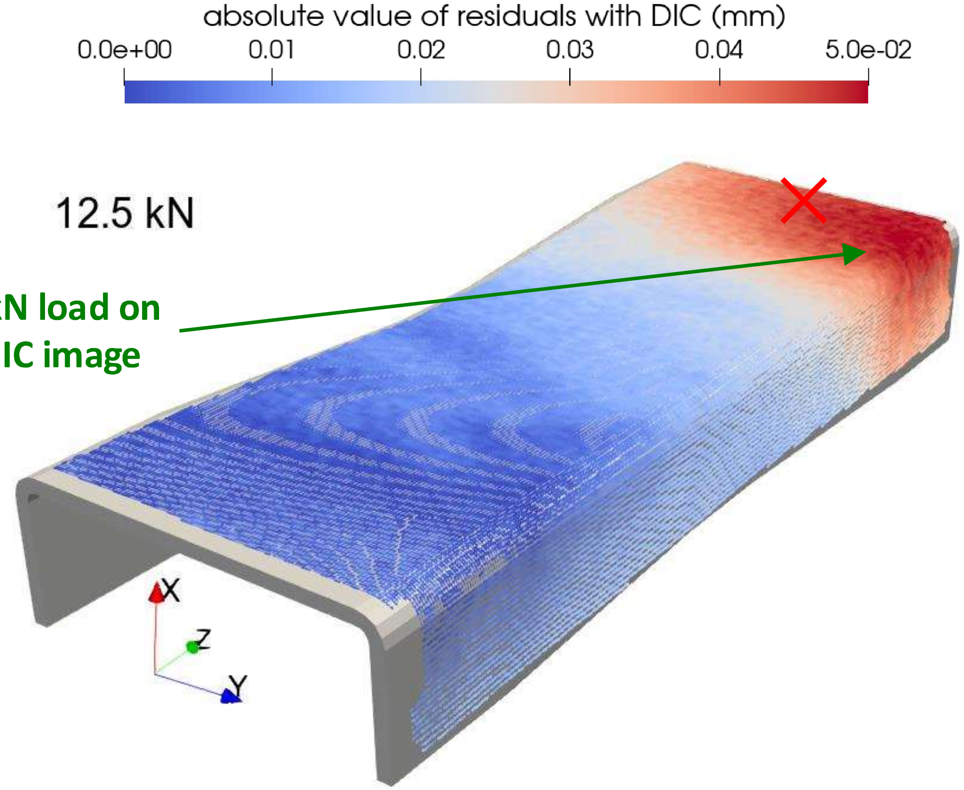
Results: Residuals with data

- Extract force-displacement at fixed location to visualise fit
- Subtract the DIC from each increment of calibrated model to visualise residuals highlighting regions of discrepancy.



Posterior prediction for point (X) on web

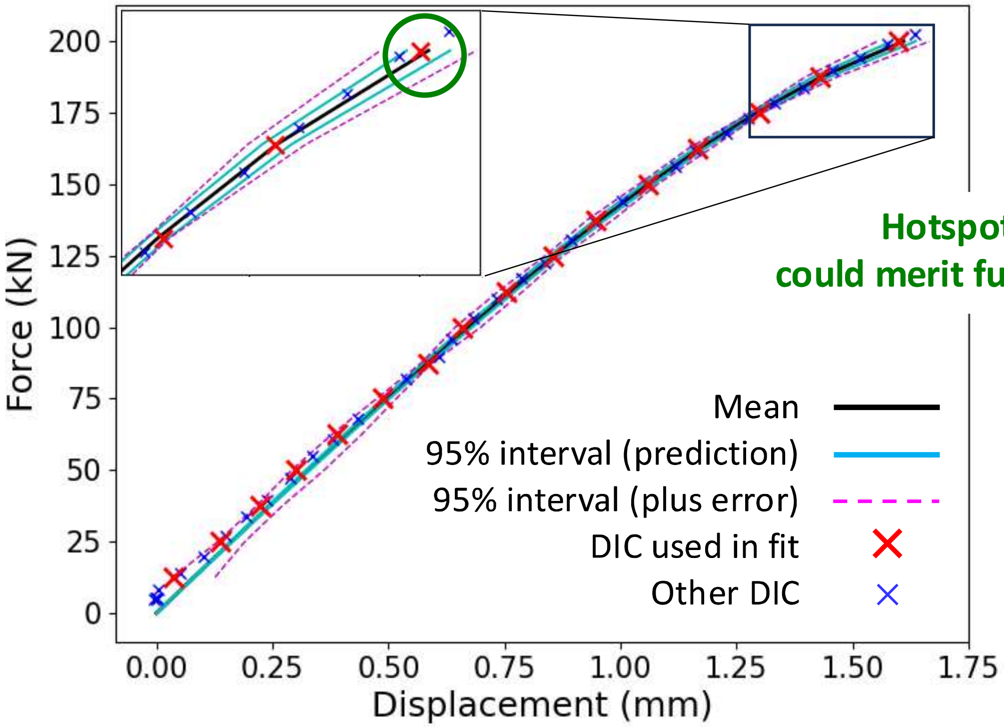
High discrepancy as 5kN load on structure for “zero” DIC image



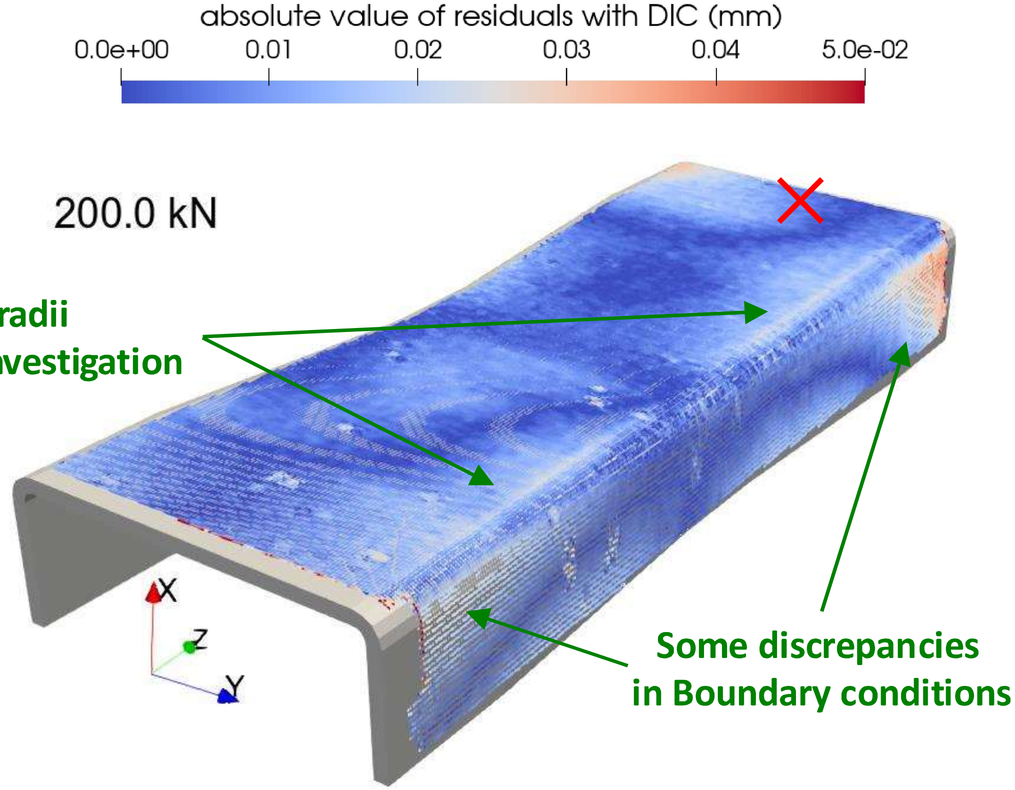
Absolute residual of mean prediction compared with DIC

Results: Residuals with data

- Extract force-displacement at fixed location to visualise fit
- Subtract the DIC from each increment of calibrated model to visualise residuals highlighting regions of discrepancy.



Posterior prediction for point (X) on web



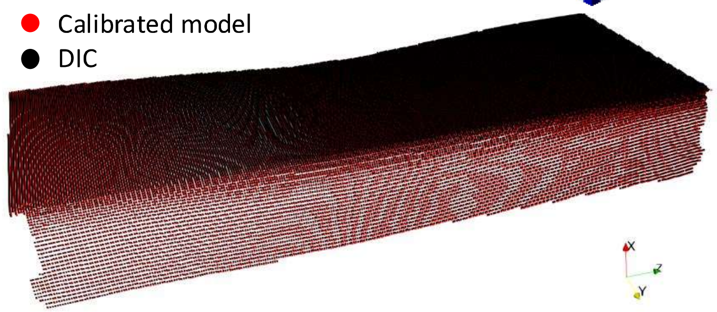
Absolute residual of mean prediction compared with DIC

Conclusions and future work

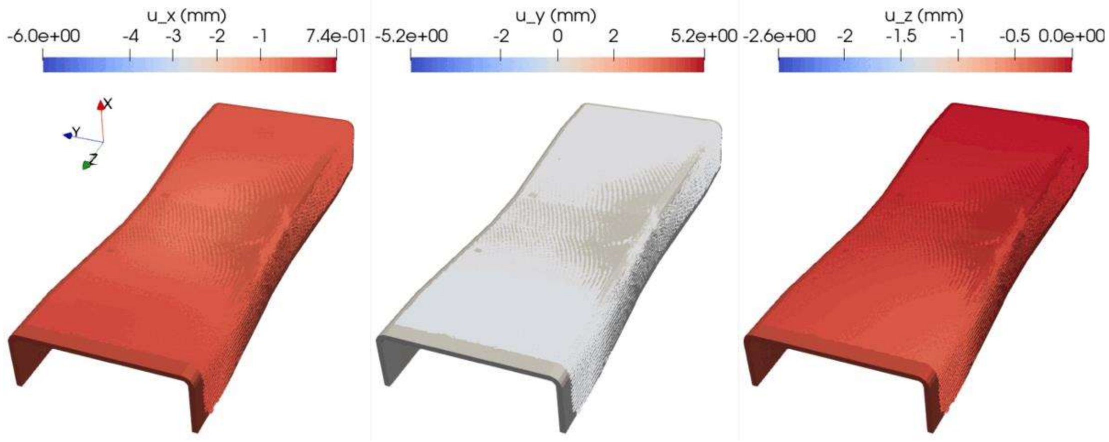
- Demonstrated a powerful statistical toolkit for comparing FE models against DIC while accounting for uncertainty in model inputs and test data.
- Overcame challenge of calibrating full-field output using high volumes of data.
- Uncertainty in boundary conditions very important.
- Highlighted manufacturing error helping with ongoing model validation.
- Future work could include:
  - Using full displacement vector.

Conclusions and future work

- Demonstrated a powerful statistical toolkit for comparing FE models against DIC while accounting for uncertainty in model inputs and test data.
- Overcame challenge of calibrating full-field output using high volumes of data.
- Uncertainty in boundary conditions very important.
- Highlighted manufacturing error helping with ongoing model validation.
- Future work could include:
  - Using full displacement vector.
  - More complex phenomena such as failure.
  - Use in model validation
  - Informing future choice of experiments.



DIC vs model point cloud with discrepancy in boundary



DIC overlaid on calibrated model predictions for historic test data

The research presented was supported by the EPSRC Programme Grant
'Certification for Design – Reshaping the Testing Pyramid'
(CerTest, EP/S017038/1)

Thank you for listening!

Any questions?